STAT/MA 41600
Practice Problems #3: November 14, 2014
Solutions by Mark Daniel Ward

1. Let X denote the number of Roseate Spoonbills in the 40 hours. Then $P(X \geq 75) = P(X \geq 74.5) = P\left(\frac{X-80}{\sqrt{80}} \geq \frac{74.5-80}{\sqrt{80}}\right) \approx P(Z \geq -0.61) = P(Z \leq 0.61) = 0.7291$.

2. Let X denote the number of errors, so $\mathbb{E}(X) = (6000)(0.04) = 240$ and $\text{Var}(X) = 240$. So $P(X < 230) = P(X < 229.5) = P\left(\frac{X-240}{\sqrt{240}} < \frac{229.5-240}{\sqrt{240}}\right) \approx P(Z < -0.68) = P(Z > 0.68) = 1 - P(Z \leq 0.68) = 1 - 0.7517 = 0.2483$.

3. Let X denote the number of crayons he checks in 40 hours, so $\mathbb{E}(X) = (295)(40) = 11{,}800$ and $\text{Var}(X) = 11{,}800$. So $P(X \geq 12{,}000) = P(X \geq 11{,}999.5) = P\left(\frac{X-11{,}800}{\sqrt{11{,}800}} \geq \frac{11{,}999.5-11{,}800}{\sqrt{11{,}800}}\right) \approx P(Z \geq 1.84) = 1 - P(Z \leq 1.84) = 1 - .9671 = 0.0329$.

4. Let X denote the number of customers, so $\mathbb{E}(X) = (8)(168) = 1344$ and $\text{Var}(X) = 1344$. So $P(1300 \leq X \leq 1400) = P(1299.5 \leq X \leq 1400.5) = P\left(\frac{1299.5-1344}{\sqrt{1344}} \leq \frac{X-1344}{\sqrt{1344}} \leq \frac{1400.5-1344}{\sqrt{1344}}\right) \approx P(-1.21 \leq Z \geq 1.54) = P(Z \leq 1.54) - P(Z < -1.21) = P(Z \leq 1.54) - P(Z > 1.21) = P(Z \leq 1.54) - (1 - P(Z \leq 1.21)) = .9382 - (1 - .8869) = .8251$.

5. Let X denote the number of Dr. Ward's errors, and let Y denote the number of his wife's errors. As in question #2, we have $\mathbb{E}(X) = (6000)(0.04) = 240$ and $\text{Var}(X) = 240$. Also $\mathbb{E}(Y) = (10{,}000)(0.025) = 250$ and $\text{Var}(Y) = 250$. So $P(X > Y) = P(X - Y > 0) = P(X - Y > 0.5) = P\left(\frac{X-Y-(240-250)}{\sqrt{240+250}} > \frac{0.5-(240-250)}{\sqrt{240+250}}\right) \approx P(Z > 0.47) = 1 - P(Z \leq 0.47) = 1 - 0.6808 = 0.3192$.