STAT/MA 41600
Practice Problems: November 14, 2014
Solutions by Mark Daniel Ward

1. Let $X_1, \ldots, X_{30}$ denote the 30 waiting times. Then $\mathbb{E}(X_j) = 1/2$ and $\operatorname{Var} X_j = 1/4$, so $\mathbb{E}(X_1 + \cdots + X_{30}) = (30)(1/2) = 15$ and $\operatorname{Var}(X_1 + \cdots + X_{30}) = (30)(1/4) = 7.5$. So $P(X_1 + \cdots + X_{30} > 14) = P(\frac{X_1+\cdots+X_{30}-15}{\sqrt{7.5}} > \frac{14-15}{\sqrt{7.5}}) \approx P(Z > -.37) = P(Z < .37) = .6443$.

2. Let $X_1, \ldots, X_{30}$ be indicator random variables that denote whether the 30 students are happy with their items, i.e., $X_j = 1$ if the jth student is happy, or $X_j = 0$ otherwise. Then $\mathbb{E}(X_j) = .60$ and $\operatorname{Var} X_j = (.60)(.40) = .24$, so $\mathbb{E}(X_1 + \cdots + X_{30}) = (30)(.60) = 18$ and $\operatorname{Var}(X_1 + \cdots + X_{30}) = (30)(.24) = 7.2$. So, using continuity correction since the X_j's are integer-valued random variables, $P(X_1 + \cdots + X_{30} \geq 20) = P(X_1 + \cdots + X_{30} \geq 19.5) = P(\frac{X_1+\cdots+X_{30}-18}{\sqrt{7.2}} \geq \frac{19.5-18}{\sqrt{7.2}}) \approx P(Z \geq .56) = 1 - P(Z < .56) = 1 - .7123 = .2877$.

3. a. We compute $\mathbb{E}(X) = \int_0^{10} x \frac{(10-x)^3}{2500}\,dx = \int_0^{10} (10-u)\frac{u^3}{2500}\,du = 2$, and $\mathbb{E}(X^2) = \int_0^{10} x^2 \frac{(10-x)^3}{2500}\,dx = \int_0^{10} (10-u)^2 \frac{u^3}{2500}\,du = 20/3$, so $\operatorname{Var}(X) = 20/3 - 2^2 = 8/3$.

b. Let $X_1, \ldots, X_{200}$ be the delays of the 200 people. So $\mathbb{E}(X_1 + \cdots + X_{200}) = (200)(2) = 400$ and $\operatorname{Var}(X_1 + \cdots + X_{200}) = (200)(8/3) = 1600/3$. So $P(X_1 + \cdots + X_{200} > 420) = P(\frac{X_1+\cdots+X_{200}-400}{\sqrt{1600/3}} > \frac{420-400}{\sqrt{1600/3}}) \approx P(Z > .87) = 1 - P(Z \leq .87) = 1 - .8078 = .1922$.

4. Let $X_1, \ldots, X_{100}$ be the completion times of the 100 people. So $\mathbb{E}(X_1 + \cdots + X_{100}) = (100)(3.5) = 350$ and $\operatorname{Var}(X_1 + \cdots + X_{100}) = (100)(1/4) = 25$. So $P(348 < X_1 + \cdots + X_{100} < 352) = P(\frac{348-350}{\sqrt{25}} < \frac{X_1+\cdots+X_{100}-350}{\sqrt{25}} > \frac{352-350}{\sqrt{25}}) \approx P(-.4 < Z < .4)$. We break this up as $P(Z < .4) - P(Z \leq -.4) = P(Z < .4) - P(Z \geq .4) = P(Z < .4) - (1 - P(Z < .4)) = 2P(Z < .4) - 1 = (2)(.6554) - 1 = .3108$.

5. We have $\mathbb{E}(X_1 + \cdots + X_{12}) = (12)(0.99) = 11.88$ and $\operatorname{Var}(X_1 + \cdots + X_{12}) = (12)(.03^2) = .0108$. So $P(Y > 1) = P(\frac{X_1+\cdots+X_{12}}{12} > 1) = P(X_1 + \cdots + X_{12} > 12) = P(\frac{X_1+\cdots+X_{12}-11.88}{\sqrt{.0108}} > \frac{12-11.88}{\sqrt{.0108}}) \approx P(Z > 1.15) = 1 - P(Z \leq 1.15) = 1 - .8749 = .1251$.