STAT/MA 41600
Practice Problems: November 14, 2014

1. Each of the 30 students in a class each orders a package. They assume that the waiting time (measured in days) for the packages are independent exponential random variables, with average waiting time of 1/2 for each package.

What is the approximate probability that the total waiting time exceeds 14 days?

2. When the students in question #1 eventually receive their packages, sometimes they are happy with the items they ordered, and sometimes they are not. Suppose that a student is happy with her/his own package with probability 0.60, independent of the happiness/unhappiness of the other students.

What is an estimate for the probability that 20 or more of the students are happy with their packages?

3. In planning for an event, the planner estimates that nobody will be on time, but nobody will be more than 10 minutes late. So he estimates that the time (in minutes) a given person will be late has density

$$f_X(x) = \frac{(10-x)^3}{2500}, \qquad \text{for } 0 \le x \le 10,$$

and $f_X(x) = 0$ otherwise.

a. Find the expected value and variance of X. Hint: It might be helpful to use the u-substitution $u = 10 - x$.

b. Estimate the probability that, among a group of 200 attendees who behave independently and follow the behavior described above, the total sum of their delay in arriving is more than 420 minutes, i.e., 7 hours.

4. Suppose that 100 marathon runners each complete a marathon in 3.5 hours, on average, with standard deviation 0.5 hours. Estimate the probability that the sum of their completion times is between 348 and 352 hours.

5. Barbara is an inspector for a water bottling company. She notices that the amount of water in each bottle has an average of 0.99 liters, and a standard deviation of 0.03 liters. She measures the quantities $X_1, \ldots, X_{12}$ in twelve independent bottles, and computes the average, Y, in these 12 bottles, i.e., $Y = \frac{X_1+\cdots+X_{12}}{12}$. Estimate the probability that $Y \geq 1$, i.e., that the average amount of water in the twelve bottles exceeds 1 liter.