STAT/MA 41600
Practice Problems: November 5, 2014

1. Let X_1, X_2, X_3 be independent exponential waiting times, each with an average of 30 minutes. Let $Y = X_1 + X_2 + X_3$.

a. What is the average (in minutes) of Y?

b. What is the standard deviation (in minutes) of Y?

2. A chef working in a kitchen believes that the waiting time until the next dessert order is exponential, with an average of 3 minutes. The times between dessert orders are assumed to be independent exponentials, also with 3 minutes on average. Let Y be the time until the next dessert order, and let Z be the subsequent time (afterwards) until the following dessert order.

[E.g., if it is 12 noon right now, and the next order arrives at 12:04 PM, and the order after that arrives at 12:11 PM, then $Y = 4$ and $Z = 7$.]

Let $X = Y + Z$. Find the density of X.

3. Suppose that the times until Hector, Ivan, and Jacob's pizzas arrive are independent exponential random variables, each with average of 20 minutes. Let X be the *sum* of the times that they spend waiting, i.e., Hector's time plus Ivan's time plus Jacob's time. Find the variance of X.

4. [Question about Exponential random variables.]

Let X be exponential with expected value 3. Let Y be another random variable that depends on X as follows: if $X > 5$, then $Y = X - 5$; otherwise, $Y = 0$.

a. Find the expected value of Y.

b. Find the variance of Y.

5. [Question about Exponential random variables.]

Suppose that Michelle, Nancy, and Olivia each are waiting for their husbands to appear. Their waiting times are assumed to be independent exponentials, and they each expect to wait 5 minutes. Let X denote the time until the very first husband appears.

What is the expected value of X? [Hint: Since X is the minimum of three independent exponential random variables, then X is exponential.]