STAT/MA 41600
Practice Problems: October 31, 2014
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1.

a. The probability is $\int_{60}^{\infty} \frac{1}{30} e^{-x/30}\,dx = -e^{-x/30}\Big|_{x=60}^{\infty} = e^{-2} = .1353.$

b. Since the average is $1/\lambda = 30$ minutes, then the variance is $1/\lambda^2 = 900$, so the standard deviation is $\sqrt{1/\lambda^2} = 30$ minutes.

2. Let X be the time (in minutes) until the next dessert; let Y be the time (in minutes) until the next appetizer. The probability is $P(X < Y) = \int_0^{\infty}\int_x^{\infty} \frac{1}{3}e^{-x/3}\frac{1}{2}e^{-y/2}\,dy\,dx = \int_0^{\infty} -\frac{1}{3}e^{-x/3}e^{-y/2}\Big|_{y=x}^{\infty}\,dx = \int_0^{\infty} \frac{1}{3}e^{-5x/6}\,dx = -\frac{1/3}{5/6}e^{-5x/6}\Big|_{x=0}^{\infty} = 2/5.$

3. Using the CDF of X, we have $P(X \leq 20) = F_X(20) = 1 - e^{-20(1/20)} = 1 - e^{-1}$. Similarly $P(Y \leq 20) = 1 - e^{-1}$ and $P(Z \leq 20) = 1 - e^{-1}$. Since the X, Y, Z are independent, then $P(\max(X, Y, Z)) = P(X \leq 20)P(Y \leq 20)P(Z \leq 20) = (1 - e^{-1})^3 = 0.2526.$

4. The company expects to pay $\int_0^3 (0)\frac{1}{1.5}e^{-x/1.5}\,dx + \int_3^{\infty}(72)(100)(x-3)\frac{1}{1.5}e^{-x/1.5}\,dx = (72)(100)\int_0^{\infty} x\frac{1}{1.5}e^{-(x+3)/1.5}\,dx = (72)(100)e^{-2}\int_0^{\infty} x\frac{1}{1.5}e^{-x/1.5}\,dx$. Notice $\int_0^{\infty} x\frac{1}{1.5}e^{-x/1.5}\,dx$ is the expected value of X, i.e., is 1.5. So the company expects to pay $(72)(100)(e^{-2})(1.5) =$ 1461.62 dollars.

5. The probability is
$\int_0^{10}\int_x^{\infty}(\frac{1}{10})(\frac{1}{5})e^{-y/5}\,dy\,dx = \int_0^{10} -\frac{1}{10}e^{-y/5}\Big|_{y=x}^{\infty}\,dx = \int_0^{10}\frac{1}{10}e^{-x/5}\,dx = \frac{1}{2}\int_0^{10}\frac{1}{5}e^{-x/5}\,dx$, but the last integral is just the CDF of an exponential with average of 5, evaluated at 10. So the overall probability is $\frac{1}{2}(1 - e^{-10/5}) = \frac{1}{2}(1 - e^{-2}) = 0.4323.$