STAT/MA 41600
Practice Problems: October 22, 2014

1. Consider a pair of random variables X, Y with constant joint density on the triangle with vertices at $(0, 0)$, $(3, 0)$, and $(0, 3)$.
a. For $0 \leq y \leq 3$, find the conditional density $f_{X|Y}(x \mid y)$ of X, given $Y = y$.

b. Find the conditional probability that $X \leq 1$, given $Y = 1$. I.e., find $P(X \leq 1 \mid Y = 1)$.

c. Find the conditional probability that $X \leq 1$, given $Y \leq 1$. I.e., find $P(X \leq 1 \mid Y \leq 1)$.

2. Consider a pair of random variables X, Y with constant joint density on the quadrilateral with vertices $(0,0)$, $(2,0)$, $(2,6)$, $(0,12)$.

a. For $0 \leq y \leq 6$, find the conditional density $f_{X|Y}(x \mid y)$ of X, given $Y = y$.

b. For $6 \leq y \leq 12$, find the conditional density $f_{X|Y}(x \mid y)$ of X, given $Y = y$.

c. Find the conditional probability that $X \leq 1$, given $3 \leq Y \leq 9$.
I.e., find $P(X \leq 1 \mid 3 \leq Y \leq 9)$.

3. Let X, Y have joint density $f_{X,Y}(x,y) = 14e^{-2x-7y}$ for $x > 0$ and $y > 0$; and $f_{X,Y}(x,y) = 0$ otherwise.

a. For $y > 0$, find the conditional density $f_{X|Y}(x \mid y)$ of X, given $Y = y$.

b. Find the conditional probability that $X \geq 1$, given $Y = 3$. I.e., find $P(X \geq 1 \mid Y = 3)$.

c. Find the conditional probability that $Y \leq 1/5$, given $X = 2.7$.
I.e., find $P(Y \leq 1/5 \mid X = 2.7)$.

4. Let X, Y have joint density $f_{X,Y}(x,y) = 18e^{-2x-7y}$ for $0 < y < x$; and $f_{X,Y}(x,y) = 0$ otherwise.

a. For $y > 0$, find the conditional density $f_{X|Y}(x \mid y)$ of X, given $Y = y$.

b. For $x > 0$, find the conditional density $f_{Y|X}(y \mid x)$ of Y, given $X = x$.

5. Suppose X, Y has joint density

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{9}(3-x)(2-y) & \text{if } 0 \le x \le 3 \text{ and } 0 \le y \le 2, \\ 0 & \text{otherwise.} \end{cases}$$

a. For $0 \le y \le 2$, find the conditional density $f_{X|Y}(x \mid y)$ of X, given $Y = y$.

b. Find the conditional probability that $X \le 2$, given $Y = 3/2$.
I.e., find $P(X \le 2 \mid Y = 3/2)$.

c. Find the conditional probability that $Y \ge 1$, given $X = 5/4$.
I.e., find $P(Y \ge 1 \mid X = 5/4)$.