STAT/MA 41600
Practice Problems: October 3, 2014
Solutions by Mark Daniel Ward

1. Hungry customers.

a. The number X of people in the survey who are eating pizza is Hypergeometric with $M = 7$, $N = 12$, and $n = 3$. So the mass of X is $p_X(x) = \frac{\binom{7}{x}\binom{5}{3-x}}{\binom{12}{3}}$.

b. The values are

$$p_X(0) = \frac{\binom{7}{0}\binom{5}{3-0}}{\binom{12}{3}} = 1/22,$$
$$p_X(1) = \frac{\binom{7}{1}\binom{5}{3-1}}{\binom{12}{3}} = 7/22,$$
$$p_X(2) = \frac{\binom{7}{2}\binom{5}{3-2}}{\binom{12}{3}} = 21/44,$$
$$p_X(3) = \frac{\binom{7}{3}\binom{5}{3-3}}{\binom{12}{3}} = 7/44.$$

c. We can write the average as $(0)(1/22) + (1)(7/22) + (2)(21/44) + (3)(7/44) = 7/4$, but it is perhaps easier to note that each person in the survey eats pizza with probability $7/12$, so the average number of survey participants eating pizza is $7/12 + 7/12 + 7/12 = (3)(7/12) = 21/12 = 7/4$.

2. Harmonicas.

a. The number X of Deluxe harmonicas is Hypergeometric with $M = 7$, $N = 19$, and $n = 8$. So $\mathbb{E}(X) = (8)(7/19) = 56/19 = 2.9474$. This can also be seen by the fact that each harmonica selected has a probability $7/19$ of being a Deluxe harmonica.

b. Since the number X of Deluxe harmonicas is Hypergeometric with $M = 7$, $N = 19$, and $n = 8$, then $\mathrm{Var}(X) = n\frac{M}{N}\left(1 - \frac{M}{N}\right)\left(\frac{N-n}{N-1}\right) = (8)(7/19)(1-7/19)(11/18) = 1232/1083 = 1.1376$.

c. Since the number X of Deluxe harmonicas is Hypergeometric with $M = 7$, $N = 19$, and $n = 8$, then $P(X = 5) = \frac{\binom{7}{5}\binom{12}{3}}{\binom{19}{8}} = 770/12597 = 0.0611$.

3. Granola bars.

a. The number X of chocolates I grab is Hypergeometric with $M = 6 + 8 = 14$, $N = 24$, and $n = 3$. So $P(X = 2) = \frac{\binom{14}{2}\binom{10}{1}}{\binom{24}{3}} = 455/1012 = 0.4496$.

b. The probability that strictly fewer than 2 are chocolate is $P(X=0)+P(X=1)=\frac{\binom{14}{0}\binom{10}{3}}{\binom{24}{3}}+\frac{\binom{14}{1}\binom{10}{2}}{\binom{24}{3}}=15/253+315/1012=375/1012=0.3706$.

c. The average number of chocolate granola bars is $n(M/N)=3(14/24)=7/4$. This can also be determined by the fact that each bar is chocolate with probability 14/24, so the average number of chocolates is $3(14/24)=7/4$.

4. Superfans.

a. The exactly probability is $\binom{60{,}000}{8}\left(\frac{1}{10{,}000}\right)^8\left(\frac{9999}{10{,}000}\right)^{59992}$.

b. If $n=60{,}000$ and $p=1/10{,}000$, then the probability in part (a) is the probability that a Binomial n,p random variable is equal to 8. Now let X be Poisson with average $\lambda=np=6$. Then $P(X=8)=\frac{e^{-6}6^8}{8!}$.

c. We have $P(X=8)=\frac{e^{-6}6^8}{8!}=0.1033$.

5. Shoppers.

a. The exact probability is

$$=\binom{100{,}000}{0}\left(\frac{49{,}999}{50{,}000}\right)^{100{,}000}+\binom{100{,}000}{1}\left(\frac{1}{50{,}000}\right)^1\left(\frac{49{,}999}{50{,}000}\right)^{99{,}999}$$
$$+\binom{100{,}000}{2}\left(\frac{1}{50{,}000}\right)^2\left(\frac{49{,}999}{50{,}000}\right)^{99{,}998}+\binom{100{,}000}{3}\left(\frac{1}{50{,}000}\right)^3\left(\frac{49{,}999}{50{,}000}\right)^{99{,}997}$$

b. If $n=100{,}000$ and $p=1/50{,}000$, then the probability in part (a) is the probability that a Binomial n,p random variable is 3 or less. Now let X be Poisson with average $\lambda=np=2$. Then $P(X\le 3)=\frac{e^{-2}2^0}{0!}+\frac{e^{-2}2^1}{1!}+\frac{e^{-2}2^2}{2!}+\frac{e^{-2}2^3}{3!}$.

c. We have $P(X\le 3)=\frac{e^{-2}2^0}{0!}+\frac{e^{-2}2^1}{1!}+\frac{e^{-2}2^2}{2!}+\frac{e^{-2}2^3}{3!}=0.8571$.