STAT/MA 41600

Practice Problems (By Request, a special Harry Potter homework): September 29, 2014

1. Quidditch Training. Hermione is frustrated because she is extremely good at spells but she is struggling to learn how to fly on her broomstick. She repeatedly tries to fly on the broomstick. Assume that her trials are independent, and a trial succeeds with probability 0.15. She conducts trials until her 4th success, and then she stops. Let X denote the number of trials that are required until (and including) her 4th success with the broomstick.

(a.) What is the mass of X?

(b.) What is the probability that it takes Hermione exactly 12 trials until her 4th success?

(c.) What is the expected number of trials until Hermione's 4th success?

2. Horcruxes. Harry Potter needs to find 7 Horcruxes to defeat You-Know-Who. Harry makes repeated attempts to guess which objects are Horcruxes. Assume that his guesses about Horcruxes are independent. Each time he guesses about a Horcrux, he is correct only 1/3 of the time. Let X be the total number of times that he makes guesses until he finds all 7 Horcruxes.

(a.) What is the expected value of X?

(b.) What is the variance of X?

(c.) What is the probability that Harry finishes his quest to find all 7 Horcruxes on his 9th guess?

3. Mandrakes. According to legend, a mandrake plant screams when it is dug up, and it will kill anyone who hears the scream. So Professor Sprout asks the students to wear earmuffs as they are digging up the plants. Assume that the students each dig up their own mandrake, one at a time. Assume that the students dig up their plants independently, one at a time, in isolation. That way, only one student risks her/his own life at a time. Also assume that each student puts her/his earmuffs properly in place with probability 0.98 and therefore has a tragic death-by-mandrake-scream with probability 0.02.

The Ministry of Magic will allow Professor Sprout to continue this risky method of teaching until three tragedies occur. After three tragedies, they will immediately intervene and force Hogwarts to close. Let X denote the number of plants that are dug up until (and including) the third tragedy.

(a.) Find the expected number of Mandrakes that are dug up before Hogwarts is forced to close, i.e., the number of Mandrakes that are dug up until (and including) the third tragedy.

(b.) Find the variance of the number of Mandrakes that are dug up before Hogwarts is forced to close, i.e., the number of Mandrakes that are dug up until (and including) the third tragedy.

4. Divination. Professor Trelawney makes many predictions, but only 12% of them come true. The accuracy of the predictions are independent. Lavender makes a bet with Ron. Lavender gets 100 galleons when Professor Trelawney's prediction is correct. Lavender loses 15 galleons when Professor Trelawney is wrong. They play this game until Professor Trelawney's 5th success, and then they stop. Let X be the number of trials until (and including) Professor Trelawney's 5th success.

(a.) Find a formula for Lavender's earnings, Y, in terms of X. [Hint: When X trials are required, then 5 of them are successes and the other $X - 5$ are failures.]

(b.) Find Lavender's expected earnings, i.e., $\mathbb{E}(Y)$.

(c.) Find the variance of Lavender's earnings, i.e., $\text{Var}(Y)$.

5. Spells. Suppose that Harry, Hermione, and Ron are each learning the Petrificus Totalus charm (i.e., the Full Body Bind). They each have probability of success 30% when they attempt the spell. Their attempts are each done completely independently (i.e., independently of their own earlier attempts; and also independently of the other people's attempts too). Harry tries the charm until his 5th success. Ron tries the charm until his 3rd success. Hermione, of course, wants to perfect the charm, so she tries the charm until her 20th success.

(a.) Let X denote the total number of attempts by all three children altogether (i.e., the sum of the number of their attempts). Find the expected value of X.

(b.) Find the variance of X.