STAT/MA 41600
Practice Problems: September 26, 2014
Solutions by Mark Daniel Ward

**1. Winnings and Losing.** (a.) Since $X$ is Geometric with probability of success 0.40, he expects to play $\mathbb{E}(X) = 1/0.40 = 2.5$ games.

(b.) Since $X$ is Geometric with probability of success 0.40, the variance of the number of games he plays is $\text{Var}(X) = 0.60/(0.40^2) = 3.75$.

(c.) The probability that he plays 4 or more games is equal to the probability that the first three games are all losses, i.e., $(.6)^3 = 0.216$.

**2. Winnings and Losing (continued).** (a.) His gain or loss is $Y = 5 + (-4)(X-1) = 9 - 4X$, since he wins 1 game and loses $X - 1$ games.]

(b.) His expected gain/loss is $\mathbb{E}(Y) = \mathbb{E}(9 - 4X) = 9 - 4\mathbb{E}(X) = 9 - 4(2.5) = -1$.

(c.) The variance of his gain/loss is $\text{Var}(Y) = \text{Var}(9 - 4X) = 16\,\text{Var}(X) = 16(3.75) = 60$.

**3. Telemarketers.** Here $X$ is Geometric with probability of success 1/8, because "success" denotes a call from the telemarketer. So $X > n$ if the first $n$ calls are unsuccessful, i.e., if the first $n$ calls are not telemarketers. So $P(X > n) = (7/8)^n$.

**4. Dating.** (a.) Since $X$ is Geometric with probability of success .07, then $\mathbb{E}(X) = 1/.07 = 100/7 = 14.29$.

(b.) Since $X$ is Geometric with probability of success .07, then $\text{Var}(X) = .93/(.07)^2 = 189.8$.

(c.) Since $X$ is memoryless, then given $X > 3$, the remaining $Y = X - 3$ people we need to call is also Geometric with probability of success .07. So the mass of $Y$ given $X > 3$ is $P(Y = 3 \mid X > 3) = (.93)^{y-1}(.07)$ for integers $y \geq 1$, and $p_Y(y) = 0$ otherwise.

**5. Hearts.** (a.) Since $X$ is Geometric with probability of success 1/4, then you expect to draw $\mathbb{E}(X) = 1/(1/4) = 4$ cards to see the first heart.

(b.) Since $X$ is memoryless, then since we are given that the first 5 cards are not hearts, it follows that the additional number of cards (after the first five are drawn) is also Geometric, with probability of success 1/4. So we expect to draw an additional $1/(1/4) = 4$ cards to see the first heart (after those first five are already drawn). (I.e., we expect that the first heart appears after 9 cards altogether.)