STAT/MA 41600
Practice Problems: September 24, 2014
Solutions by Mark Daniel Ward

1. Winnings and Losing. (a.) His expected total gain/loss is $\mathbb{E}(9X - 40) = 9\mathbb{E}(X) - 40 = 9(4) - 40 = -4$.

(b.) The variance of his total gain or loss is $\text{Var}(9X - 40) = 81\,\text{Var}(X) = (81)(10)(.6)(.4) = 194.4$.

(c.) His winnings are $9X - 40$, so the probability that he wins \$32 or more is

$$\begin{aligned}
P(9X - 40 \geq 32) &= P(9X \geq 72) = P(X \geq 8) \\
&= \binom{10}{8}(.4)^8(.6)^2 + \binom{10}{9}(.4)^9(.6)^1 + \binom{10}{10}(.4)^{10}(.6)^0 \\
&= 45\left(\frac{2304}{9765625}\right) + 10\left(\frac{1536}{9765625}\right) + 1\left(\frac{1024}{9765625}\right) \\
&= \frac{120064}{9765625} \\
&= 0.012295.
\end{aligned}$$

2. Telemarketers. (a.) Since X is Binomial with $n = 3$ and $p = 1/8$, then

$$p_X(x) = \binom{3}{x}(1/8)^x(7/8)^{3-x} \qquad \text{when } x \text{ is } 0,\ 1,\ 2, \text{ or } 3,$$

and $p_X(x) = 0$ otherwise. So

$$p_X(0) = \frac{343}{512}, \qquad p_X(1) = \frac{147}{512}, \qquad p_X(2) = \frac{21}{512}, \qquad p_X(3) = \frac{1}{512}.$$

3. Dating. (a.) Let X be the number of people who accept the invitation. So X is Binomial with $n = 20$ and $p = .07$. So the probability of $X \geq 3$ is

$$\begin{aligned}
P(X \geq 3) &= 1 - P(X = 0) - P(X = 1) - P(X = 2) \\
&= 1 - \binom{20}{0}(.07)^0(.93)^{20} - \binom{20}{1}(.07)^1(.93)^{19} - \binom{20}{2}(.07)^2(.93)^{18} \\
&= 1 - (1)(0.2342) - (20)(0.01763) - (190)(0.001327) \\
&= 0.161
\end{aligned}$$

(b.) Since X is Binomial$(n = 20,\ p = .07)$, then $\mathbb{E}(X) = np = (20)(.07) = 1.40$.

(c.) Since X is Binomial$(n = 20,\ p = .07)$, then $\text{Var}(X) = npq = (20)(.07)(.93) = 1.3020$.

4. Dining Hall. (a.) Since X, Y, Z are independent Binomials with the same p, then $X + Y + Z$ is Binomial too, with $n = 7 + 7 + 7 = 21$ and with the same $p = .65$.

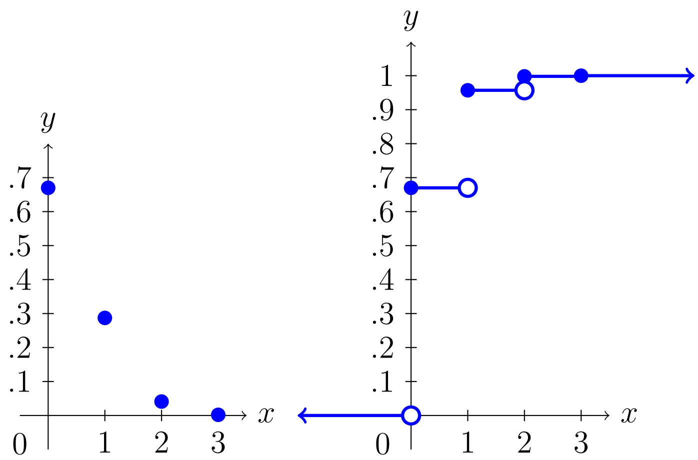

Figure 1: For X telemarketers, the mass $p_X(x) = P(X = x)$ and CDF $F_X(x) = P(X \leq x)$

Since $X + Y + Z$ is Binomial with $n = 21$ and $p = .65$, then $\text{Var}(X + Y + Z) = npq = (21)(.65)(.35) = 4.7775$.

5. Hearts. (a.) Let X_j denote the jth card that is drawn. Then $\mathbb{E}(X_j) = 13/52 = 1/4$. So $\mathbb{E}(X) = \mathbb{E}(X_1 + \cdots + X_7) = \mathbb{E}(X_1) + \cdots + \mathbb{E}(X_j) = 1/4 + \cdots + 1/4 = 7/4 = 1.75$.

(b.) No, X is not binomial because the X_j's are dependent.