1. Butterflies. We have $\mathbb{E}(X) = \mathbb{E}(X_1 + X_2 + X_3) = \mathbb{E}(X_1) + \mathbb{E}(X_2) + \mathbb{E}(X_3) = .17 + .25 + .45 = .87$.

2. Dependence/independence among dice rolls. We have

$$\mathbb{E}(X) = \mathbb{E}\left(\sum_{j=1}^{\infty} X_j\right) = \sum_{j=1}^{\infty} \mathbb{E}(X_j) = \sum_{j=1}^{\infty} (5/6)^{j-1} = \frac{1}{1 - \frac{5}{6}} = 6$$

The same steps show $\mathbb{E}(Y) = \mathbb{E}\left(\sum_{j=1}^{\infty} Y_j\right) = \sum_{j=1}^{\infty} \mathbb{E}(Y_j) = \sum_{j=1}^{\infty} (5/6)^{j-1} = \frac{1}{1-\frac{5}{6}} = 6$.

3. Wastebasket basketball. We have

$$\begin{aligned}
\mathbb{E}(X) &= \mathbb{E}(X_1 + X_2 + X_3 + X_4 + X_5 + X_6) \\
&= \mathbb{E}(X_1) + \mathbb{E}(X_2) + \mathbb{E}(X_3) + \mathbb{E}(X_4) + \mathbb{E}(X_5) + \mathbb{E}(X_6) \\
&= 1 + \left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^4 + \left(\frac{2}{3}\right)^5 = \frac{665}{243} = 2.7366
\end{aligned}$$

4. Two 4-sided dice. We have

$$\mathbb{E}(X) = \mathbb{E}(X_1 + X_2 + X_3 + X_4) = \mathbb{E}(X_1) + \mathbb{E}(X_2) + \mathbb{E}(X_3) + \mathbb{E}(X_4) = 1 + \left(\frac{3}{4}\right)^2 + \left(\frac{2}{4}\right)^2 + \left(\frac{1}{4}\right)^2 = 15/8$$

5. Pick two cards. We have $\mathbb{E}(X) = \mathbb{E}(X_1 + X_2) = \mathbb{E}(X_1) + \mathbb{E}(X_2) = 12/52 + 12/52 = 6/13$.

Before looking at the cards, put one in your left hand and one in your right hand. Let Y_1 and Y_2 indicate, respectively, whether the cards in your left and right hands (respectively) are 10's. Then $Y = Y_1 + Y_2$. Find the expected value of Y by finding the expected value of the sum of the indicator random variables.

We have $\mathbb{E}(Y) = \mathbb{E}(Y_1 + Y_2) = \mathbb{E}(Y_1) + \mathbb{E}(Y_2) = 4/52 + 4/52 = 2/13$.