STAT/MA 41600
Practice Problems: September 12, 2014
Solutions by Mark Daniel Ward

1. Butterflies. Let A, B, C be, respectively, the events that Alice, Bob, Charlie finds a butterfly. Then

$$p_X(0) = P(A^c)P(B^c)P(C^c) = 0.3424$$
$$p_X(1) = P(A)P(B^c)P(C^c) + P(A^c)P(B)P(C^c) + P(A^c)P(B^c)P(C) = 0.4644$$
$$p_X(2) = P(A)P(B)P(C^c) + P(A)P(B^c)P(C) + P(A^c)P(B)P(C) = 0.1741$$
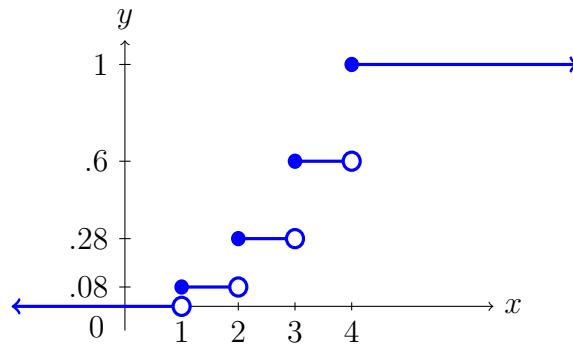$$p_X(3) = P(A)P(B)P(C) = 0.0191$$

2. Appetizers.

Figure 1: the CDF $F_X(x)$ for the price X of a randomly-chosen appetizer.

3. Wastebasket basketball. The mass of X is

$$p_X(1) = 1/3 = .3333$$
$$p_X(2) = (2/3)(1/3) = 2/9 = .2222$$
$$p_X(3) = (2/3)^2(1/3) = 4/27 = .1481$$
$$p_X(4) = (2/3)^3(1/3) = 8/81 = .0988$$
$$p_X(5) = (2/3)^4(1/3) = 16/243 = .0658$$
$$p_X(6) = 1 - 1/3 - 2/9 - 4/27 - 8/81 - 16/243 = 32/243 = .1317$$
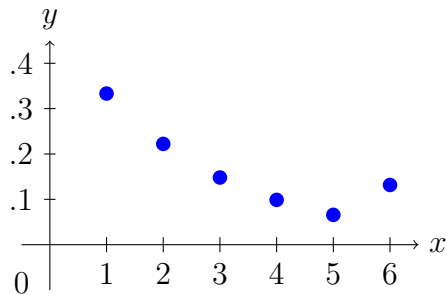
So the mass looks like

Figure 2: the mass $p_X(x)$ for the number X of attempts required for a basket.
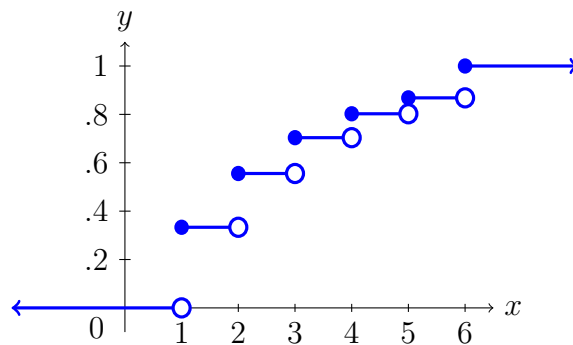
Draw the CDF of X.

Figure 3: the CDF $F_X(x)$ for the number X of attempts required for a basket.

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4. Two 4-sided dice. The mass of X is

$$p_X(2) = 1/16 = .0625$$
$$p_X(3) = 2/16 = .125$$
$$p_X(4) = 3/16 = .1875$$
$$p_X(5) = 4/16 = .25$$
$$p_X(6) = 3/16 = .1875$$
$$p_X(7) = 2/16 = .125$$
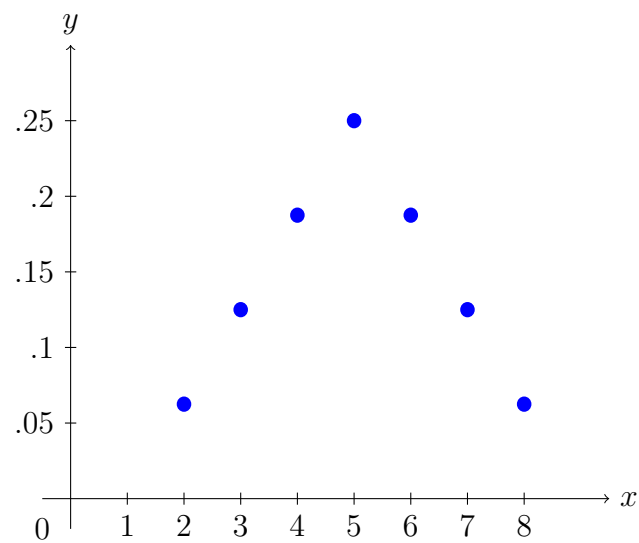$$p_X(8) = 1/16 = .0625$$

So the mass looks like

Figure 4: the mass $p_X(x)$ for the sum X of two 4-sided dice.
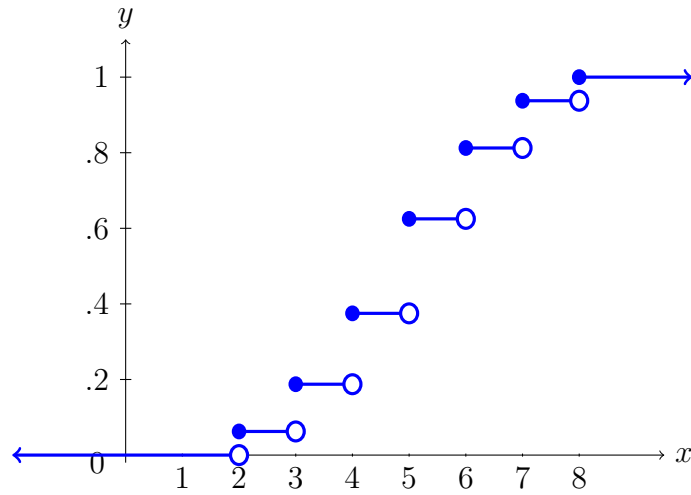
Draw the CDF of X.

Figure 5: the CDF $F_X(x)$ for the sum X of two 4-sided dice.

5. Pick two cards. As discussed in Problem Set 7, the mass of X is:

$$p_X(0) = 30/51 = .5882, \qquad p_X(1) = 80/221 = .3620, \qquad p_X(2) = 11/221 = .0498.$$
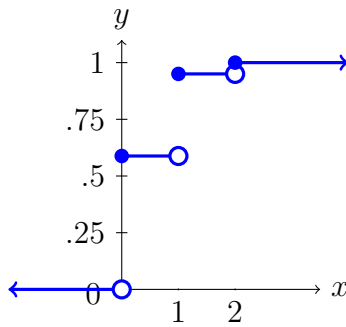
So the CDF of X is

Figure 6: the CDF $F_X(x)$ for the number X of face cards.