STAT/MA 41600
Practice Problems: September 10, 2014
Solutions by Mark Daniel Ward

1. Harmonicas. Since X is a waiting time, then X takes value in the interval $[0, \infty)$, so X is a continuous random variable.

Since Y is a nonnegative integer, i.e., Y takes values 0, 1, 2, 3, ..., then Y is a discrete random variable.

2. Choosing a page at random.
(a.) Find $P(X = 122)$. We have $P(X = 122) = 1/1000$.
(b.) Find $P(X = 977)$. We have $P(X = 977) = 1/1000$.
(c.) Find $P(X = -2)$. We have $P(X = -2) = 0$.
(d.) Find $P(X = 1003)$. We have $P(X = 1003) = 0$.
(e.) When x is an integer between 0 and 999, find $P(X = x)$. We have $P(X = x) = 1/1000$.
(f.) Find $P(X \leq 3)$. We have $P(X \leq 3) = 4/1000$.
(g.) Find $P(X \leq 122)$. We have $P(X \leq 122) = 123/1000$.
(h.) Find $P(12 \leq X \leq 17)$. We have $P(12 \leq X \leq 17) = 6/1000$.
(i.) Find $P(X > 122)$. We have $P(X > 122) = 1 - P(X \leq 122) = 1 - 123/1000 = 877/1000$.
(j.) Find $P(X = 15.73)$. We have $P(X = 15.73) = 0$.
(k.) Find $P(X \leq 15.73)$. We have $P(X \leq 15.73) = 16/1000$.

3. Gloves. a. When j is a positive integer, $P(X = j) = (\frac{4}{5})^{j-1}(\frac{1}{5})$.
b. When j is a positive integer with $1 \leq j \leq 5$, then $P(X = j) = 1/5$.

4. Three dice. Since the sum of the three dice is an integer between 3 and 18, then $P(X = j)$ is strictly positive for integers j with $3 \leq j \leq 18$.

5. Pick two cards. Let A_1, A_2 be (respectively) the events that the first, second card is a face card. Even though the cards appear simultaneously, we can just randomly treat one of them as the first and the other as the second. So

$$P(X = 0) = P(A_1^c \cap A_2^c) = P(A_1^c)P(A_2^c \mid A_1^c) = (40/52)(39/51) = 30/51.$$

Use the same A_1, A_2 as above. Then

$$P(X = 1) = P(A_1 \cap A_2^c) + P(A_1^c \cap A_2) = (12/52)(40/51) + (40/52)(12/51) = 80/221.$$

Use the same A_1, A_2 as above. Then

$$P(X = 2) = P(A_1 \cap A_2) = (12/52)(11/51) = 11/221.$$