Transformation of a random variable

Say X is a continuous Uniform random variable on $[0,5]$.

Define $Y = \pi X^2$ i.e. if we treat X as the radius of a circle, then Y is the area of the circle.

Two ways to get $E(Y)$.

① $E(Y) = E(\pi X^2) = \int_0^5 (\pi x^2)\left(\frac{1}{5}\right) dx = \frac{25\pi}{3}$

② Find CDF of Y, then density of Y, then expected value of Y.
(We learn more about Y this way.)

$$F_Y(a) = P(Y \le a) = P(\pi X^2 \le a) = P\left(X \le \sqrt{\frac{a}{\pi}}\right) = \frac{\sqrt{\frac{a}{\pi}} - 0}{5-0}$$

↖ know $X \ge 0$ so do not need the negative part here

for $0 < a < \pi 5^2 = 25\pi$

$$f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy}\left(\frac{1}{5}\sqrt{\frac{y}{\pi}}\right) = \frac{1}{10\sqrt{\pi y}}$$

$$E(Y) = \int_0^{25\pi} (y) \frac{1}{10\sqrt{\pi y}}\, dy = \frac{25\pi}{3} \quad \text{as before!}$$