Moment Generating Function of a Binomial random variable X with parameters n and p.

$$M_X(t) = E(e^{tX}) = \sum_x e^{tx} p_X(x) = \sum_{x=0}^{n} e^{tx} \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} \binom{n}{x} (e^t p)^x (1-p)^{n-x}$$

$$= (e^t p + 1 - p)^n$$

use Binomial THM:

$$\sum_{j=0}^{n} \binom{n}{j} a^j b^{n-j} = (a+b)^n$$

use x instead of j

$e^t p$ instead of a

$1-p$ instead of b

$$E(X) = \frac{\partial}{\partial t} M_X(t) \Big|_{t=0} = M_X'(0)$$

$= np$ which is $E(X)$ as we know.

$$E(X^2) = \frac{\partial^2}{\partial t^2} M_X(t) \Big|_{t=0} = M_X''(0)$$

$$= (n)(n-1)p^2 + np$$

$$Var(X) = E(X^2) - (E(X))^2 = (n)(n-1)p^2 + np - (np)^2$$

$= np(1-p)$ as we know ✓