Generating Functions

$$g(t) = \sum_{j=0}^{\infty} \frac{\boxed{c_j}}{j!} t^j = \frac{c_0}{0!} t^0 + \frac{c_1}{1!} t^1 + \frac{c_2}{2!} t^2 + \frac{c_3}{3!} t^3 + \cdots$$

This is the generating function for the series of numbers $c_0, c_1, c_2, c_3, \ldots$

Notice, if we differentiate j times, we can get c_j back:

$$c_j = g^{(j)}(0) = \frac{\partial^j}{\partial t^j} g(t) \Big|_{t=0}$$

In probability, we are particularly interested in moment generating functions

The moment generating function of a random variable X is:

$$M_X(t) = \sum_{j=0}^{\infty} \frac{E(X^j)}{j!} t^j = E\left(\sum_{j=0}^{\infty} \frac{(tX)^j}{j!} \right) = E\left[e^{tX} \right]$$

t is an input to the function, and
X is a random variable.

$E(X^j) =$ expected value of X^j
$=$ jth moment of X e.g. 1st moment is $E(X)$
2nd moment is $E(X^2)$
.....

Idea is that we wrap up all moments $E(X^j)$ as the coefficients of $\frac{t^j}{j!}$.