Poisson splitting Not exactly related, but closely related.

Suppose we have N items, where N is a Poisson random variable with $E(N)=\lambda$.
i.e. the parameter of N is λ.

Also suppose, regardless of N's value, each of the items is split into exactly one of several categories, and the splittings are independent, and occur with the same probabilities. Then the number of each type of item is independent from the numbers of other types, and each type has a Poisson number of members, with parameter $p\lambda$ where p is the probability of becoming that type.

E.g. Suppose a Poisson number of people go to the post office, with mean 20 people. Suppose each person is 60% likely to be a female or 40% male, classification is independent.
Then the number of males at the post office is Poisson with mean $(.40)(20) = 8$
and the number of females is also Poisson with mean $(.60)(20) = 12$
and these counts are independent.
Emphasize even if we know that there are (say) 11 men at the post office, the number of females is still independent and still Poisson with mean 12.

How does this fit with what we just discussed?
Let $X_1, X_2, \ldots, X_N$ be indicators for whether the first, second, ..., Nth persons are male.

$$E(X_1 + \ldots + X_N) = \sum_n E(X_1 + \ldots + X_n \mid N=n) P(N=n)$$
$$= \sum_n (n)(.40) P(N=n)$$
$$= (.40) \sum_n n P(N=n)$$
$$= (.40)(20)$$
$$= 8.$$