__Example__ Suppose the amount of tuition students owe is independent, and suppose each student owes, on average, 5,000.

Suppose we pick a random number of students, with 20 students picked on average. What is the average amount of tuition that the selected students owe, altogether? (Assuming the # of students picked is independent from the amounts they owe.)

Let Y denote the number of students picked. Once $Y=y$ is known, let $X_1, \dots, X_y$ be the tuition amounts.

$$\underset{\substack{\uparrow\\ \text{random}}}{E(X_1+\dots+X_Y)} = \underset{\substack{\uparrow\\ \text{exp. value over } Y=y\text{'s}}}{E}\big(\underbrace{E(X_1+\dots+X_y|}_{\text{is exp value over } X\text{'s}} Y=y)\big)$$

$$= E(\underbrace{5000+5000+\dots+5000}_{5000y} | Y=y)$$

$$= E(5000Y) = 5000E(Y) = (5000)(20)$$

$$= 100{,}000.$$

More generally, if the X_j's have the same expected values

$$E(X_1+\dots+X_y | Y=y) = yE(X_1)$$

$$E(X_1+\dots+X_Y) = \underset{\substack{\uparrow\\ \text{still random}}}{Y}\underbrace{E(X_1)}_{\text{a number}}$$

$$\underset{\substack{\uparrow\\ \text{with respect to } Y.}}{E}\big(\underbrace{E(X_1+\dots+X_Y}_{\text{with respect to } X\text{'s}}|Y)\big) = E(Y\cdot E(X_1)) = E(X_1)E(Y)$$

If Y is integer valued:

$$E(E(X_1+\dots+X_Y|Y)) = \sum_y E(X_1+\dots+X_y|Y=y)P(Y=y)$$

$$= \sum_y yE(X_1)P(Y=y)$$ if the X's are identically distributed, or its enough if they have the same exp. values

$$= E(X_1)\sum_y yP(Y=y)$$

$$= E(X_1)E(Y).$$