Example Suppose X and Y have joint density $f_{X,Y}(x,y) = 40e^{-3x-5y}$ for $0<x<y$

$= 0$ otherwise

Find the expected value of X, given $Y = y$.

First, we need $f_{X|Y}(x|y) = \dfrac{f_{X,Y}(x,y)}{f_Y(y)} = \dfrac{40e^{-3x-5y}}{\frac{40}{3}(e^{-5y}-e^{-8y})}$ for $0<x<y$

$f_Y(y)$ ↖

$$= \int_0^y 40e^{-3x-5y}\,dx = \frac{40}{3}(e^{-5y}-e^{-8y})$$

$$E(X|Y=y) = \int_0^y (X)\,\frac{40e^{-3x-5y}}{\frac{40}{3}(e^{-5y}-e^{-8y})}\,dx = \frac{1}{3} - \frac{y}{e^{3y}-1}$$

notice y is fixed

For instance

$$E(X|Y=2) = \int_0^2 (X)\,\frac{40e^{-3x-5(2)}}{\frac{40}{3}(e^{-5(2)}-e^{-8(2)})}\,dx = \frac{1}{3} - \frac{2}{e^{3(2)}-1} = \underline{0.3284}$$