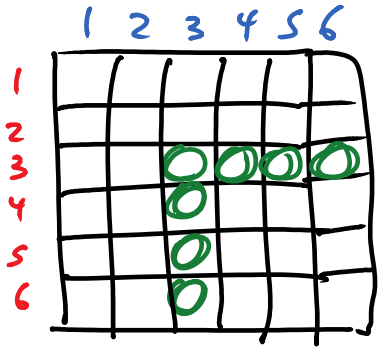
Example Roll two fair six-sided dice.

Let X denote the maximum value.

Let Y denote the minimum value.

Find $E(X \mid Y=3)$



$$p_{X|Y}(x|3) = \begin{cases} \frac{1}{7} & x=3 \\ \frac{2}{7} & x=4 \\ \frac{2}{7} & x=5 \\ \frac{2}{7} & x=6 \\ 0 & \text{otherwise} \end{cases}$$

$$E(X \mid Y=3) = (3)p_{X|Y}(3|3) + (4)p_{X|Y}(4|3) + (5)p_{X|Y}(5|3) + (6)p_{X|Y}(6|3)$$

$$= (3)(1/7) + (4)(2/7) + (5)(2/7) + (6)(2/7)$$

(as always with conditional probability mass functions, they must sum to 1: $1/7 + 2/7 + 2/7 + 2/7 = 1$)

$$= \frac{33}{7}$$