Comparison of the definition and use of variances vs covariances

Variance of X is defined as

$$\begin{aligned} Var(X) &= E\left((X-E(X))^2\right) \\ &= E\left((X-E(X))(X-E(X))\right) \end{aligned}$$

Usage of $Var(X)$ is often

$$\begin{aligned} Var(X) &= E\left((X-E(X))(X-E(X))\right) \\ &= E\left(X^2 - XE(X) - E(X)\cdot X + (E(X))^2\right) \\ &= E(X^2) - (E(X))^2 - (E(X))^2 + (E(X))^2 \\ &= E(X^2) - (E(X))^2 \end{aligned}$$

Covariance of X and Y is defined as

$$Cov(X,Y) = E\left((X-E(X))(Y-E(Y))\right)$$

Notice if we use $Y=X$ then

$$Var(X) = Cov(X,X).$$

Also, we do not need X, Y to be independent, to apply the concept of $Cov(X,Y)$.

Similarly, for covariances often use

$$\begin{aligned} Cov(X,Y) &= E\left((X-E(X))(Y-E(Y))\right) \\ &= E\left(XY - XE(Y) - YE(X) + E(X)E(Y)\right) \\ &= E(XY) - E(X)E(Y) - E(X)E(Y) + E(X)E(Y) \\ &= E(XY) - E(X)E(Y) \end{aligned}$$

again notice $Cov(X,X) = Var(X)$

Also this is the form in our earlier discussion about variance of a sum of random variables.