Central Limit Theorem

Suppose $X_1, X_2, \ldots$ is an infinite sequence of independent random variables (not necessarily Normal) that each have mean μ and variance σ^2. Fix "a" then the probability that the sum of the first n of the random variables, suitably scaled and shifted, is less than a will go to the same probability that a standard Normal random variable is less than a (as $n \to \infty$).

$$\lim P\left(\frac{X_1 + \cdots + X_n - n\mu}{\sqrt{n\sigma^2}} \leq a \right) = \int_{-\infty}^{a} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz = F_Z(a)$$

The Central Limit Theorem is a limiting result.
In practice, we often apply it to finite but large collections of independent random variables, to model their sum by a standard Normal random variable.

Two caveats: The CLT works better and better, the larger n is, and there is no cutoff for which it works completely above some particular n.

We must be sure to get the scaling and shifting correct, if we are going to apply the CLT properly.