Laws of Large Numbers

Weak Law of Large Numbers: Fix $\varepsilon > 0$ (usually small, e.g. $\varepsilon = \frac{1}{10000}$) and consider an infinite sequence of random variables $X_1, X_2, X_3, \ldots$ that are independent. Then the probability that the average of the first n random variables is more than ε away from the mean of the random variables converges to 0 as $n \to \infty$. (Need the X_j's to all share a common expected value, say μ.)

$$\lim_{n\to\infty} P\left(\left| \frac{X_1 + \cdots + X_n}{n} - \mu \right| < \varepsilon \right) = 1$$

i.e. $$\lim_{n\to\infty} P\left(\left| \frac{X_1 + \cdots + X_n}{n} - \mu \right| \geq \varepsilon \right) = 0$$

Strong Law of Large Numbers

Again, assume $X_1, X_2, X_3, \ldots$ is an infinite sequence of independent random variables with mean μ. The strong law of large numbers says the average of the first n of the random variables will converge as $n \to \infty$ to the mean μ, with probability 1.

$$P\left(\lim_{n\to\infty} \frac{X_1 + \cdots + X_n}{n} = \mu \right) = 1.$$

Need higher math, especially to prove the Strong Law of Large Numbers in its full generality.