Sum of Independent Normal Random Variables

If X is a normal random variable with expected value μ_X and standard deviation σ_X then $rX+s$ is also a normal random variable (think: we essentially just scale and shift the units of X).

$$E(rX+s) = rE(X)+s = r\mu_X + s$$

$$Var(rX+s) = r^2\, Var(X) = r^2 \sigma_X^{\,2}$$

So the standard deviation of $rX+s$ is $r\sigma_X$.

Nice fact: If $X_1, X_2, \ldots, X_n$ are <u>independent</u> Normal random variables with means $\mu_1, \mu_2, \ldots, \mu_n$ respectively and standard deviations $\sigma_1, \sigma_2, \ldots, \sigma_n$ respectively, then $X_1 + X_2 + \cdots + X_n$ is a normal random variable as well.

The mean of $X_1 + X_2 + \cdots + X_n$ is $\mu_1 + \mu_2 + \cdots + \mu_n$

Because the X_j's are independent, the variance of $X_1 + X_2 + \cdots + X_n$ is $\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_n^2$ and the standard deviation is $\sqrt{\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_n^2}$.

If the X_j's are independent Normal random variables with the same mean μ and the same variance σ^2 then $X_1 + \cdots + X_n$ has mean $n\mu$ and variance $n\sigma^2$ so the standard deviation is $\sqrt{n\sigma^2}$.