Variance of a Beta random variable. Assume $\alpha = 3$, $\beta = 8$

$f_X(x) = 360\, x^2 (1-x)^7$ for $0 < x < 1$

$$E(X^2) = \int_0^1 (x^2)(360)\, x^2 (1-x)^7\, dx \qquad \begin{array}{l} u = 1-x \\ x = 1-u \end{array} \quad du = -dx$$

$$= \int_1^0 360\,(1-u)^4 u^7 (-1)\, du$$

$$= 360 \int_0^1 (1 - 4u + 6u^2 - 4u^3 + u^4) u^7\, du$$

$$= 360 \int_0^1 (u^7 - 4u^8 + 6u^9 - 4u^{10} + u^{11})\, du$$

$$= 360 \left(\frac{u^8}{8} - \frac{4}{9} u^9 + \frac{6}{10} u^{10} - \frac{4}{11} u^{11} + \frac{u^{12}}{12} \right) \Big|_{u=0}^{1}$$

$$= 360 \left(\frac{1}{8} - \frac{4}{9} + \frac{6}{10} - \frac{4}{11} + \frac{1}{12} \right) = 360 \left(\frac{1}{3960} \right) = \frac{1}{11}$$

$$\mathrm{Var}(X) = E(X^2) - \left(E(X)\right)^2 = \frac{1}{11} - \left(\frac{3}{11}\right)^2 = \frac{2}{121}$$ as claimed earlier.