Example Suppose $\alpha = 3$, $\beta = 8$.

Then X has density $f_X(x) = \begin{cases} \frac{(\alpha+\beta-1)!}{(\alpha-1)!(\beta-1)!} x^{\alpha-1}(1-x)^{\beta-1} & \text{for } 0<x<1 \\ 0 & \text{otherwise} \end{cases}$

$$f_X(x) = \frac{10!}{2!\,7!} x^2 (1-x)^7 \quad \text{for } 0<x<1$$

Find the CDF of X.

$F_X(x) = 0$ for $x \le 0$

$F_X(x) = 1$ for $x \ge 1$

$$\frac{10!}{2!\,7!} = \frac{(10)(9)(8)}{2} = \frac{720}{2} = 360$$

for $0 < a < 1$

$$F_X(a) = \int_0^a f_X(x)\,dx = \int_0^a 360 x^2 \underbrace{(1-x)^7}_{\text{could expand, but painful}} dx$$

Hint: If necessary, do u-sub so that x is raised to the larger power and $1-x$ to the smaller power.

$u = 1-x \qquad x = 1-u \qquad du = -dx$

$$F_X(a) = \int_1^{1-a} 360(1-u)^2 u^7 (-1)\,du = \int_{1-a}^1 360(1-2u+u^2)u^7\,du$$

$$= \int_{1-a}^1 360(u^7 - 2u^8 + u^9)\,du$$

$$= 360\left(\frac{u^8}{8} - \frac{2u^9}{9} + \frac{u^{10}}{10}\right)\Bigg|_{u=1-a}^{1}$$

$$= 360\left[\left(\frac{1}{8} - \frac{2}{9} + \frac{1}{10}\right) - \left(\frac{(1-a)^8}{8} - \frac{2(1-a)^9}{9} + \frac{(1-a)^{10}}{10}\right)\right]$$