Example: Suppose X has density $f_X(x) = \frac{7^3}{(3-1)!} x^{3-1} e^{-7x}$ for $x > 0$

$= 0$ otherwise

and suppose Y has density $f_Y(y) = \frac{7^5}{(5-1)!} y^{5-1} e^{-7y}$ for $y > 0$

$= 0$ otherwise

and suppose X and Y are independent. Q: What kind of random variable is $X + Y$??

Notice that X has the same distribution as $X_1 + X_2 + X_3$ where the X_j's are independent Exponential random variables, each with $\lambda = 7$.

Notice that Y has the same distribution as $Y_1 + Y_2 + Y_3 + Y_4 + Y_5$ where the Y_j's are independent Exponential random variables, each with $\lambda = 7$.

So $X + Y$ has the same distribution as the sum of 8 independent Exponential random variables, each with $\lambda = 7$

So $X + Y$ is a Gamma random variable with $\lambda = 7$, $r = 8$.

More generally if we sum several independent Gamma random variables with a common parameter λ and possibly different r values, then the sum is also a Gamma random variable with the same parameter λ and with r equal to the sum of the r's of the Gamma random variables that make up the sum.