Gamma random variables

The idea of a Gamma random variable is to sum together a finite (fixed) number of independent exponential random variables.

We have seen a concept like this before, when we studied discrete random variables. Think back to when we learned about Geometric random variables. Remember that if (for example) X_1, X_2, X_3, X_4, X_5 are Geometric random variables that are independent, then $X = X_1 + X_2 + X_3 + X_4 + X_5$, then X is a Negative Binomial random variable. Remember that X was the number of trials until the 5th success occurs. In general, remember that a Negative Binomial random variable is the number of trials until the rth success, where r is some fixed number. This gives some context to the motivation for Gamma random variables.

Following this same kind of example, let's suppose that X_1, X_2, X_3, X_4, X_5 are exponential random variables that are i independent. Then their sum $X = X_1+X_2+X_3+X_4+X_5$ is called a Gamma random variable with parameter $r = 5$. Just like with Geometric versus Negative Binomial random variables, we need a second parameter too, i.e., we need to know the value of the parameter λ that comes from the Exponential random variables, and moreover, this parameter λ has to be the same for all of the independent Exponential random variable that we sum to build our Gamma random variable.

So if $X_1, \ldots, X_5$ are independent Exponential random variables all with a common parameter λ (say, for instance, $\lambda = 3$), then $X = X_1 + \cdots + X_5$ is a Gamma random variable with parameters $\lambda = 3$ and $r = 5$. The r tells the number of random variables we are summing up.

Think of Gamma random variables as a sum of waiting times. Remember that an Exponential random variable is often a waiting time for something to occur. For instance, suppose that each X_j is the waiting time until the next email arrives in our inbox. Then the X we defined above, i.e., the sum of 5 independent X_j's would be the time until the 5th email arrives.

This is the general idea too. A Gamma random variable is often thought of as the time until the rth occurrence of some kind of repeating event, where the times between consecutive events are independent Exponential random variables.