Minimum of a whole collection of independent exponential random variables. Let $X_1$, $X_2$,..., $X_n$ be independent exponential random variables with $E(X_1) = 1/\lambda_1$, $E(X_2) = 1/\lambda_2$, ..., $E(X_n) = 1/\lambda_n$. Now let's define $Z$ as the minimum of this whole collection of exponential random variables. Since we already discussed the $n = 2$ case, i.e., if $Z$ is the minimum of two independent exponential random variables we know $Z$ would be exponential as well, we might guess that $Z$ turns out to be an exponential random variable in this more general case, i.e., no matter what $n$ we use. For instance, if $Z$ is the minimum of 17 independent exponential random variables, should $Z$ still be an exponential random variable? The answer is yes! Why? The reasoning is similar to what we saw before: Suppose $a > 0$; then

$$\begin{aligned}
P(Z > a) &= P(\min(X_1, X_2, \ldots, X_n) > a) \\
&= P(X_1 > a, X_2 > a, \ldots, X_n > a) \\
&= P(X_1 > a)P(X_2 > a) \cdots P(X_n > a) \qquad \text{because } X_1, X_2, \ldots, X_n \text{ are independent} \\
&= e^{-a\lambda_1} e^{-a\lambda_2} \cdots e^{-a\lambda_n} \\
&= e^{-a(\lambda_1+\lambda_2+\cdots+\lambda_n)}
\end{aligned}$$

and just to make sure that you see it, the CDF of $Z$ is, for $a > 0$,

$$F_Z(a) = P(Z \le a) = 1 - P(Z > a) = 1 - e^{-a(\lambda_1+\lambda_2+\cdots+\lambda_n)}.$$

So indeed $Z$ is an exponential random variable with parameter $\lambda_1 + \lambda_2 + \cdots + \lambda_n$. So the density of $Z$ is

$$f_Z(z) = (\lambda_1 + \lambda_2 + \cdots + \lambda_n)e^{-z(\lambda_1+\lambda_2+\cdots+\lambda_n)},$$

for $z > 0$, and $f_Z(z) = 0$ for $z \le 0$.