Minimum of two independent exponential random variables: Suppose that X and Y are independent exponential random variables with $E(X) = 1/\lambda_1$ and $E(Y) = 1/\lambda_2$. Let $Z = \min(X, Y)$. Something neat happens when we study the distribution of Z, i.e., when we find out how Z behaves. First of all, since $X > 0$ and $Y > 0$, this means that $Z > 0$ too. So the density $f_Z(z)$ of Z is 0 for $z < 0$. Now let's focus on the density $f_Z(z)$ of Z, for $z > 0$. Easier to find the CDF of Z. In fact, even easier to find the complement of the CDF of Z.

If we want $P(Z > a)$ for some $a > 0$, we just want

$$P(Z > a) = P(\min(X, Y) > a) = P(X > a,\ Y > a) = P(X > a)P(Y > a) = e^{-a\lambda_1}e^{-a\lambda_2} = e^{-a(\lambda_1+\lambda_2)}.$$

So

$$F_Z(a) = 1 - P(Z > a) = 1 - e^{-a(\lambda_1+\lambda_2)}.$$

So the CDF of Z has exactly the form of the CDF of an exponential random variable, so Z itself is exponential, with $E(Z) = 1/(\lambda_1 + \lambda_2)$. Also

$$f_Z(z) = (\lambda_1 + \lambda_2)e^{-(\lambda_1+\lambda_2)z},$$

for $z > 0$, and $f_Z(z) = 0$ otherwise.

So the short of the story is that Z is an exponential random variable with parameter $\lambda_1 + \lambda_2$, i.e., $E(Z) = 1/(\lambda_1 + \lambda_2)$.

Hint: This will not work if you are trying to take the maximum of two independent exponential random variables, i.e., the maximum of two independent exponential random variables is not itself an exponential random variable. This nice property only happens with the minimum of independent exponential random variables.