Memoryless property of exponential random variables. Resembles the memoryless property of geometric random variables. The idea is that we start with (and often use) the fact that, if X is exponential with $E(X) = 1/\lambda$, then $P(X > a) = e^{-\lambda a}$ for $a > 0$.

The idea of the memoryless properly, for example, is that

$$P(X > 14 \mid X > 8) = P(X > 6)$$

Intuitively, thinking about the fact that exponential random variables are waiting times, this translates into saying that the probability we have to wait 14 minutes or more, given that we already waited at least 8 minutes, is equal to the (unconditional, i.e., not assuming anything) probability that we wait 6 minutes or more from the start. It is essentially like starting over, after having waited those first (given) 8 minutes, without our "thing" occurring, where the "thing" is whatever we are waiting for.

To calculate this,

$$P(X > 14 \mid X > 8) = \frac{P(X > 14 \text{ and } X > 8)}{P(X > 8)} = \frac{P(X > 14)}{P(X > 8)} = \frac{e^{-14\lambda}}{e^{-8\lambda}} = e^{-6\lambda} = P(X > 6).$$

Moreover, this idea works in complete generality. Nothing special at all about 6, 8, and 14. If we want to know the probability we have to wait $a + b$ or more minutes altogether, and if we have already waited a minutes so far, then this is equal to the probability that (starting from the beginning) we just have to wait b minutes or more. That's the intuitive way of seeing this. Formally,

$$P(X > a + b \mid X > a) = P(X > b),$$

for a, b positive constants. Why does this hold? Justification is similar to what we saw before. We have

$$P(X > a+b \mid X > a) = \frac{P(X > a + b \text{ and } X > a)}{P(X > a)} = \frac{P(X > a + b)}{P(X > a)} = \frac{e^{-(a+b)\lambda}}{e^{-a\lambda}} = e^{-b\lambda} = P(X > b).$$

This is the memoryless property of exponential random variables. Warning: Do not expect this to work for all continuous random variables! Exponential random variables are the only kind of continuous random variable where this works, i.e., exponential random variables are the only continuous random variables with the memoryless property.