Suppose X is an exponential random variable with density

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find $E(X) = \int_0^\infty x \lambda e^{-\lambda x}\, dx = \underbrace{(x)(-e^{-\lambda x})\Big|_{x=0}^{\infty}}_{} - \int_0^\infty -e^{-\lambda x}\, dx$

focus on this part

$= 0$ for $x = 0$

for $x \to \infty$

$\to -\frac{\infty}{\infty}$ written as $\frac{-x}{e^{\lambda x}}$

need L'Hospital's rule

$= \lim_{x \to \infty} \frac{-1}{(e^{\lambda x})(\lambda)} = 0$

use int. by parts.

$u = x \qquad du = dx$

$dv = \lambda e^{-\lambda x} dx \qquad v = \frac{\lambda e^{-\lambda x}}{-\lambda} = -e^{-\lambda x}$

$$E(X) = \int_0^\infty e^{-\lambda x}\, dx$$

$$= \frac{e^{-\lambda x}}{-\lambda}\Big|_{x=0}^{\infty} = \frac{1}{\lambda} \quad \text{so } \boxed{E(X) = \frac{1}{\lambda}}$$

$$E(X^2) = \int_0^\infty x^2 \lambda e^{-\lambda x}\, dx = \underbrace{(x^2)(-e^{-\lambda x})\Big|_{x=0}^{\infty}}_{= 0 - 0 = 0} - \underbrace{\int_0^\infty (2x)(-e^{-\lambda x})\, dx}$$

int. by parts.

$u = x^2 \qquad du = 2x\, dx$

$dv = \lambda e^{-\lambda x} dx \qquad v = \frac{\lambda e^{-\lambda x}}{-\lambda} = -e^{-\lambda x}$

$= \frac{2}{\lambda} \int_0^\infty x \lambda e^{-\lambda x}\, dx$

put in λ

divid by λ

$= \frac{2}{\lambda} \cdot E(X) = \frac{2}{\lambda} \cdot \frac{1}{\lambda} = \frac{2}{\lambda^2}$

$$\mathrm{Var}(X) = E(X^2) - (E(X))^2 = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{1}{\lambda^2}$$

Also standard deviation of X is $1/\lambda$.