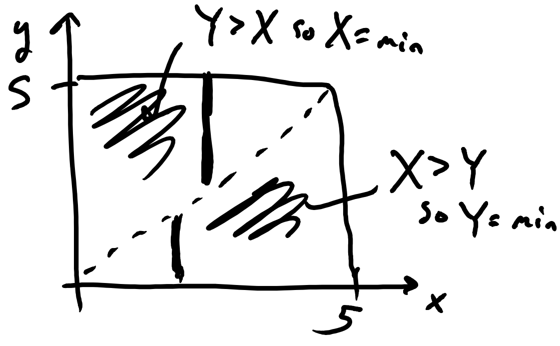
Example Let X, Y be independent continuous uniform random variables, each on $[0,5]$. Let $Z = \min(X,Y)$. Find $E(Z)$.

Two ways. One way:

$$E(\min(X,Y)) = \int_0^5\int_0^x \underbrace{\min(x,y)}_{=y} \cdot \underbrace{\frac{1}{25}}_{=\frac{1}{5}\cdot\frac{1}{5}=f_{X,Y}(x,y)}\, dy\, dx + \int_0^5\int_x^5 \underbrace{\min(x,y)}_{=x} \cdot \underbrace{\frac{1}{25}}_{\frac{1}{5}\cdot\frac{1}{5}=f_{X,Y}(x,y)}\, dy\, dx$$



Evaluate integrals.

$$\int_0^x y\,\frac{1}{25}\,dy = \frac{y^2}{2}\,\frac{1}{25}\Big|_{y=0}^{x} = \frac{x^2}{50}$$

$$\int_0^5 \frac{x^2}{50}\,dx = \frac{x^3}{50\cdot 3}\Big|_{x=0}^{5} = \frac{5^3}{50\cdot 3} = \frac{5}{6}$$

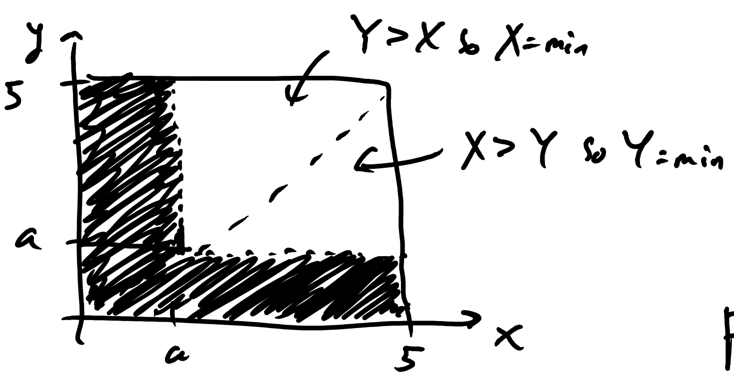
Similarly second integral is $5/6$ too

$$\int_0^5\int_x^5 x\cdot\frac{1}{25}\,dy\,dx = \frac{5}{6} \text{ also.}$$

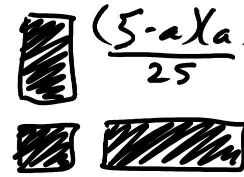
So $E(\min(X,Y)) = E(Z) = \frac{5}{6} + \frac{5}{6} = \frac{5}{3}$.

Second method: Find CDF of Z. Know $0 \le Z \le 5$

When is $Z \le a$ (for $0 < a < 5$)??



$P(Z \le a) = P((X,Y) \in \text{shaded region})$



$$= \frac{(5-a)(a)}{25} + \frac{a^2}{25} + \frac{(5-a)(a)}{25}$$

$$= \frac{10a - a^2}{25} = F_Z(a)$$

$$F_Z(z) = \frac{10z - z^2}{25}$$

$$f_Z(z) = \frac{10-2z}{25}$$

$$E(Z) = \int_0^5 z\cdot\frac{10-2z}{25}\,dz$$

$$= \int_0^5 \frac{10z - 2z^2}{25}\,dz$$

$$= \left(\frac{10z^2}{50} - \frac{2z^3}{75}\right)\Big|_{z=0}^{5}$$

$$E(Z) = \frac{10\cdot 5^2}{50} - \frac{2\cdot 5^3}{75}$$

$$= \frac{10}{2} - \frac{2\cdot 5}{3}$$

$$= \frac{30-20}{6} = \frac{10}{6} = \frac{5}{3}$$

So $\boxed{E(Z) = 5/3.}$