Example Suppose X is a continuous uniform random variable on $[5,45]$.

The density of X must be $f_X(x) = \begin{cases} \frac{1}{45-5} = \frac{1}{40} & \text{for } 5<x<45 \\ 0 & \text{otherwise} \end{cases}$

$$E(X) = \frac{5+45}{2} = \frac{50}{2} = 25.$$

$$Var(X) = \frac{(45-5)^2}{12} = \frac{40^2}{12} = \frac{20^2}{3} = 400/3.$$

What about probabilities?

$$P(X \le 22) = \int_5^{22} \frac{1}{40}\,dx = \frac{x}{40}\Big|_{x=5}^{22} = \frac{22-5}{40} = \frac{17}{40}$$

alternative method $= \frac{\text{length of } [5,22]}{\text{length of } [5,45]} = \frac{17}{40}$, no integration is needed.

i.e. no integration needed because integrating a constant $\left(\frac{1}{40}\right)$ over an interval $([5,22])$ so get $\left(\frac{1}{40}\right)(17) = \frac{17}{40}$.

Also, conditional probabilities are continuous uniforms, i.e have constant densities. E.g. If we condition the X above on being >10, i.e. we condition on $X>10$, we essentially replacing the continuous uniform on $(5,45)$ with a new continuous uniform $(10,45]$.

So, e.g., $P(X>19 \mid X>10) = \frac{P(X>19 \,\&\, X>10)}{P(X>10)} = \frac{P(X>19)}{P(X>10)}$

$$= \frac{\text{length of } [19,45] / \cancel{\text{length of } [5,45]}}{\text{length of } [10,45] / \cancel{\text{length of } [5,45]}}$$

$$= \frac{\text{length of } [19,45]}{\text{length of } [10,45]}$$

$$= 26/35.$$

Alt. view $P(X>19 \mid X>10) = \frac{P(X>19)}{P(X>10)} = \frac{\int_{19}^{45} \frac{1}{40}\,dx}{\int_{10}^{45} \frac{1}{40}\,dx} = \frac{(\cancel{\frac{1}{40}})(45-19)}{(\cancel{\frac{1}{40}})(45-10)} = \frac{26}{35}$.

This gives us a natural to double check answers, without integrating.

Works in general, e.g. if X is unif on $[a,b]$ and then we are given $X>c$ for some $c \in [a,b]$ we essentially replace X by a new unif random variable on the interval $[c,b]$. I.e. we are just changing the interval where the continuous uniform random variable is defined.