Expected value of a continuous uniform random variable X: We know X has constant density $f_X(x) = 1/(b-a)$ on some interval $[a,b]$. So

$$E(X) = \int_a^b (x)\frac{1}{b-a}\,dx = \frac{1}{b-a}\frac{x^2}{2}\Big|_{x=a}^{b} = \frac{1}{b-a}\frac{b^2-a^2}{2} = \frac{1}{b-a}\frac{(b-a)(b+a)}{2} = \frac{a+b}{2}.$$

This makes intuitive sense, because the density is constant (evenly spread) across the finite length interval $[a,b]$, so we might guess that the expected value would be directly in the middle of this interval, and indeed it is.

What about $E(X^2)$?

$$E(X^2) = \int_a^b (x^2)\frac{1}{b-a}dx = \frac{1}{b-a}\frac{x^3}{3}\Big|_{x=a}^{b} = \frac{1}{b-a}\frac{b^3-a^3}{3} = \frac{1}{b-a}\frac{(b-a)(a^2+ab+b^2)}{3} = \frac{a^2+ab+b^2}{3}.$$

Now we can find the variance of X:

$$Var(X) = E(X^2) - (E(X))^2 = \frac{a^2+ab+b^2}{3} - \left(\frac{a+b}{2}\right)^2 = \frac{(b-a)^2}{12}.$$

(Here we just used 12 as a common denominator and simplified. Please check.)