Now let's show that

$$Var(aX + b) = a^2 Var(X).$$

This is for a, b constants. We already know this for discrete random variables. Same kind of idea works, but just want to remember this.

$$\begin{aligned}
Var(aX + b) &= E((aX + b)^2) - (E(aX + b))^2 \\
&= E(a^2X^2 + 2abX + b^2) - (aE(X) + b)(aE(X) + b) \\
&= a^2E(X^2) + 2abE(X) + b^2 - a^2(E(X))^2 - 2abE(X) - b^2 \\
&= a^2E(X^2) - a^2(E(X))^2 \\
&= a^2(E(X^2) - (E(X))^2) \\
&= a^2Var(X)
\end{aligned}$$

Another nice fact: If X and Y are independent, then $Var(X + Y) = Var(X) + Var(Y)$. Why?

$$\begin{aligned}
Var(X + Y) &= E((X + Y)^2) - (E(X + Y))^2 \\
&= E(X^2 + 2XY + Y^2) - (E(X) + E(Y))^2 \\
&= E(X^2) + 2E(X)E(Y) + E(Y^2) - (E(X))^2 - 2E(X)E(Y) - (E(Y))^2 \\
&\qquad\qquad \text{using } E(XY) = E(X)E(Y) \text{ at the start since } X, Y \text{ independent} \\
&= E(X^2) - (E(X))^2 + E(Y^2) - (E(Y))^2 \\
&= Var(X) + Var(Y)
\end{aligned}$$

Another nice fact: Apply that rule over and over again, and for independent $X_1, X_2, \ldots, X_n$, we have:

$$Var(X_1 + \cdots + X_n) = Var(X_1) + \cdots + Var(X_n),$$

where we caution that $X_1, \ldots, X_n$ must be independent to apply this rule.

We can also put this rule together with the one at the top of the page, to get

$$\begin{aligned}
Var(a_1X_1 + \cdots + a_nX_n) &= Var(a_1X_1) + \cdots + Var(a_nX_n) \\
&= a_1^2Var(X_1) + \cdots + a_n^2Var(X_n),
\end{aligned}$$

where we are assuming here that $X_1, \ldots, X_n$ are independent (this must be known, to use this rule), and that $a_1, \ldots, a_n$ are constants.

Another common mistake is to try to use this rule with $X_1, \ldots, X_n$ not independent. We need independent to apply this rule. One last common mistake is that occasionally people forget that the $a_1, \ldots, a_n$ should be constants, not random variables themselves. They must be constants to apply this kind of rule.