Example: Suppose that X has density $f_X(x) = (1/4)x^3$ for $0 < x < 2$, and $f_X(x) = 0$ otherwise. First check that this is a valid density.

$$\int_0^2 (1/4)x^3\,dx = (1/4)x^4/4|_{x=0}^2 = (1/4)2^4/4 = 1.$$

So this is indeed a valid probability density function.

Now let's find the expected value of X:

$$E(X) = \int_0^2 (x)(1/4)x^3/dx = 1/4\int_0^2 x^4\,dx = (1/4)x^5/5|_{x=0}^2 = (1/4)2^5/5 = 8/5.$$

Now what about the expected value of X^2?

$$E(X^2) = \int_0^2 x^2(1/4)x^3/dx = 1/4\int_0^2 x^5\,dx = (1/4)x^6/6|_{x=0}^2 = (1/4)2^6/6 = 16/6 = 8/3.$$

Finally, let's compute the variance of X:

$$Var(X) = E(X^2) - (E(X))^2 = 8/3 - (8/5)^2 = ((8)(25) - (64)(3))/75 = (200 - 192)/75 = 8/75.$$

Then the standard deviation is just the square root of the variance, so $\sigma_X = \sqrt{8/75} = (2/5)\sqrt{2/3}$.