The expected value of a function of a continuous random variable, and also the variance of a continuous random variable. First, if g is a function, to find $E(g(X))$, where X is a continuous random variable, we compute

$$E(g(X)) = \int_{-\infty}^{\infty} g(x) f_X(x)\, dx.$$

For instance, if $g(X) = X^2$, then

$$E(g(X)) = E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx.$$

As with discrete random variables, we define the variance of a continuous random variable to be

$$Var(X) = E((X - \mu)^2),$$

where $\mu = E(X)$. As with discrete random variables, we often use

$$\begin{aligned} Var(X) &= E((X-\mu)^2) \\ &= \int_{-\infty}^{\infty} (x-\mu)^2 f_X(x)\, dx \\ &= \int_{-\infty}^{\infty} (x^2 - 2x\mu + \mu^2) f_X(x)\, dx \\ &= \int_{-\infty}^{\infty} x^2 f_X(x)\, dx - 2\mu \int_{-\infty}^{\infty} x\, dx + \mu^2 \int_{-\infty}^{\infty} f_X(x)\, dx \\ &= E(X^2) - 2\mu\mu + \mu^2 \\ &= E(X^2) - \mu^2 \\ &= E(X^2) - (E(X))^2 \end{aligned}$$

This is what we might expect, since we also had $Var(X) = E(X^2) - (E(X))^2$ with discrete random variables.

One more thing: As with discrete random variables, the standard deviation of a random variable is just square root of the variance,

$$\sigma_X = \sqrt{Var(X)}.$$

The variance and the standard deviation both measure how spread out a random variable is from the mean (from the center). The nice thing about the standard deviation is that it has the same units as the random variable itself, e.g., if X is given in terms of miles, then the standard deviation is too.