Another example, suppose X has constant density on the range $[0,5]$. since the integral of the density must be 1, this constant has to be $1/5$. In other words,

$$\int_0^5 f_X(x)\,dx = \int_0^5 1/5\,dx = 1.$$

What is the expected value of X?

$$E(X) = \int_0^5 (x)(1/5)\,dx = (1/5)(x^2/2)|_{x=0}^5 = (1/5)(5^2/2) = 5/2.$$

This is just a special case of a more general idea: Suppose instead that X has a constant density on the range $[a,b]$. Then

$$\int_a^b f_X(x)\,dx = \int_a^b 1/(b-a)\,dx = 1.$$

So the constant of the density has to be $1/(b-a)$. No other constant will work. So $f_X(x) = 1/(b-a)$ for $a < x < b$, and otherwise $f_X(x) = 0$. What is the expected value of such a random variable?

$$E(X) = \int_a^b (x)(1/(b-a))\,dx = (1/(b-a))x^2/2|_{x=a}^b = (1/(b-a))(b^2-a^2)/2 = (1/(b-a))(b-a)(b+a)/2$$

So altogether we just get

$$E(X) = (b+a)/2.$$

FYI, in our example above, at the top, we got $E(X) = 5/2$, and this matches with $(5+0)/2$ in the special case with $a = 0$ and $b = 5$. This works much more generally than that too.