Suppose we study an example where X has density $f_X(x) = 5e^{-5x}$ for $x > 0$ and $f_X(x) = 0$ otherwise. Let's find the expected value of X.

$$E(X) = \int_0^\infty (x)(5e^{-5x})\,dx$$

We need u-substitution. We use $u = 5x$ and $dv = e^{-5x}\,dx$. So $du = 5dx$, and $v = \frac{e^{-5x}}{-5}$.

So

$$E(X) = (5x)\frac{e^{-5x}}{-5}|_{x=0}^{\infty} - \int_0^\infty \frac{e^{-5x}}{-5} 5\,dx$$

Now let's check the first part. I claim it is 0. Why? The $x = 0$ part of the first term is obviously 0. The ∞ part is $\lim_{x\to\infty}(5x)\frac{e^{-5x}}{-5} = \lim_{x\to\infty} -\frac{x}{e^{5x}} = \lim_{x\to\infty} -\frac{1}{5e^{5x}} = 0$, by L'Hospital's Rule. The essential idea is that, as $x \to \infty$, the e^{5x} in the denominator grows large much much faster than the x from the numerator. So the first term in the $E(X)$ was 0 altogether. Now if we multiply and divide by 5 in the second term, we have simplified things to:

$$E(X) = \int_0^\infty e^{-5x}\,dx = \frac{e^{-5x}}{-5}|_{x=0}^{\infty} = 0 - (-1/5) = 1/5.$$

More generally, now let's consider X with density $f_X(x) = \lambda e^{-\lambda x}$ for $x > 0$ and $f_X(x) = 0$ otherwise.

$$E(X) = \int_0^\infty (x)(\lambda e^{-\lambda x})\,dx$$

Again use u-substition with $u = \lambda x$, and $dv = e^{-\lambda x}\,dx$. So $du = \lambda dx$ and $v = \frac{e^{-\lambda x}}{-\lambda}$. So

$$E(X) = (\lambda x)\frac{e^{-\lambda x}}{-\lambda} - \int_0^\infty \frac{e^{-\lambda x}}{-\lambda}\lambda\,dx$$

Again the first term is 0 altogether, and we simplify things to:

$$E(X) = \int_0^\infty e^{-\lambda x}\,dx = \frac{e^{-\lambda x}}{-\lambda}|_{x=0}^{\infty} = 0 - (-1/\lambda) = 1/\lambda.$$