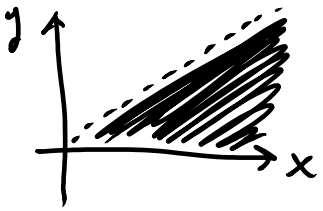
Example Say X, Y have joint density $f_{X,Y}(x,y) = 9e^{-3x}$ for $0<y<x$
$= 0$ otherwise



We found $f_X(x)$ earlier:

$$f_X(x) = \int f_{X,Y}(x,y)\,dy = \int_0^x 9e^{-3x}\,dy = 9e^{-3x}y\Big|_{y=0}^{x} = 9xe^{-3x}$$

So $f_{Y|X}(y|x) = \dfrac{f_{X,Y}(x,y)}{f_X(x)} = \dfrac{9e^{-3x}}{9xe^{-3x}}$ for $0<y<x$

$= \dfrac{1}{x}$ for $0<y<x$

For instance, if $X=3$ is given, then the conditional density of Y, given $X=3$ is $f_{Y|X}(y|3) = \frac{1}{3}$ for $0<y<3$. Check $\int_0^3 f_{Y|X}(y|3)\,dy = \int_0^3 \frac{1}{3}\,dy = 1$ ✓

Now calculate density of Y.

$$f_Y(y) = \int f_{X,Y}(x,y)\,dx = \int_y^\infty 9e^{-3x}\,dx = \frac{9e^{-3x}}{-3}\Big|_{x=y}^{\infty} = \frac{9e^{-3y}}{3} = 3e^{-3y}$$ for $0<y<x$.

So the conditional density of X, given $Y=y$, is

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} = \frac{9e^{-3x}}{3e^{-3y}} = 3e^{-3x+3y}$$ for $0<y<x$

For example if $Y=2$ is given, then the conditional density of X, given $Y=2$, is $f_{X|Y}(x|2) = 3e^{-3x+6}$ for $2<x$

Check: $\int_2^\infty f_{X|Y}(x|2)\,dx = \int_2^\infty 3e^{-3x+6}\,dx = \frac{3e^{-3x+6}}{-3}\Big|_{x=2}^{\infty} = e^{-3(2)+6} = e^0 = 1$ ✓

$P(Y\le 1 \mid X=3) = \int_0^1 \frac{1}{3}\,dy = \frac{1}{3}$ $\quad P(1.7\le Y\le 2.9 \mid X=3) = \int_{1.7}^{2.9} \frac{1}{3}\,dx = \frac{1.2}{3}$

$P(3\le X\le 4 \mid Y=2) = \int_3^4 3e^{-3x+6}\,dx = \frac{3e^{-3x+6}}{-3}\Big|_{x=3}^{4} = e^{-3(3)+6} - e^{-3(4)+6} = e^{-3} - e^{-6}$
