Some notes about conditional densities:

The value of y is considered as "fixed" when we are working with a conditional density. For instance, suppose we want to study the distribution of X when Y is known to have value 1.73. Then we are looking at

$$f_{X|Y}(x|1.73).$$

We might ask questions, for instance, as we would for other continuous random variables, e.g., might ask what is $P(2 \leq X \leq 7 \mid Y = 1.73)$? To find the solution, we integrate the conditional density of X, given $Y = 1.73$, over the interval $2 \leq x \leq 7$:

$$P(2 \leq X \leq 7 \mid Y = 1.73) = \int_2^7 f_{X|Y}(x|1.73)\, dx = \int_2^7 \frac{f_{X,Y}(x, 1.73)}{f_Y(1.73)}\, dx.$$

When X and Y are independent, then remember that the joint density of X and Y factors into the product of the two densities of the two random variables. So, in such a case,

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} = \frac{f_X(x) f_Y(y)}{f_Y(y)} = f_X(x),$$

in other words, when X and Y are independent, the conditional density of X given $Y = y$ is just exactly the same as the (unconditional) density of X, saying nothing at all about Y.