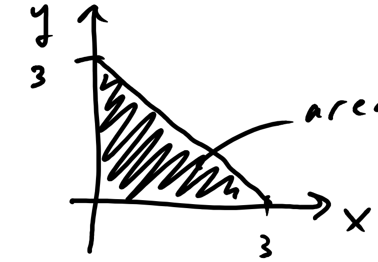
Example Suppose X, Y have joint density $f_{X,Y}(x,y) = \frac{2}{9}$ for $x>0, y>0, x+y \le 3$

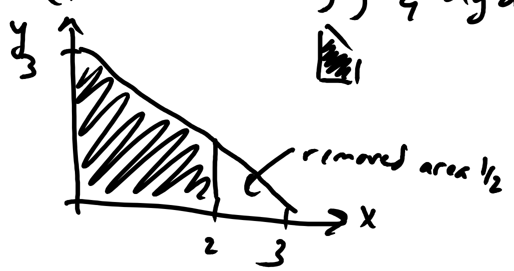
$f_{X,Y}(x,y) = 0$ otherwise



So $\iint \frac{2}{9}\, dy\,dx = \frac{2}{9}\cdot(\text{area of }) = \left(\frac{2}{9}\right)\left(\frac{9}{2}\right) = 1.$

$P(X \le 2) = \iint \frac{2}{9}\, dy\,dx = \frac{2}{9}(\text{area of })$

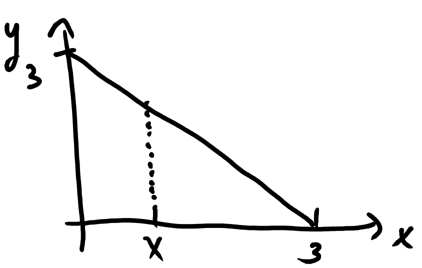
$= \frac{2}{9}\cdot\frac{8}{2} = \boxed{\frac{8}{9}}$



Q: Are X, Y indep? No, because the joint density is not defined to be nonzero in a rectangular grid or series of rectangular grids.

What is the density of X? (Symmetric: Know the density of Y is similar.)

$f_X(x) = \int f_{X,Y}(x,y)\,dy$ — if $0 > x$ or $3 < x$, then $f_X(x) = 0$ because $0 \le X \le 3$.



For $0 < x < 3$,

$f_X(x) = \int_0^{3-x} \frac{2}{9}\,dy = \frac{2}{9}y\Big|_{y=0}^{3-x} = \frac{2}{9}(3-x).$

Check that this a valid density:

$f_X(x) = \begin{cases} \frac{2}{9}(3-x) & \text{for } 0<x<3 \\ 0 & \text{otherwise} \end{cases}$

Always nonnegative. Does it integrate to 1?

$\int_0^3 \frac{2}{9}(3-x)\,dx = \frac{2}{9}\left(3x - \frac{x^2}{2}\right)\Big|_{x=0}^{3} = \frac{2}{9}\left(9 - \frac{9}{2}\right) = \left(\frac{2}{9}\right)\left(\frac{9}{2}\right) = 1.$ ✓

By symmetry, we get the density of Y too:

$f_Y(y) = \frac{2}{9}(3-y)$ for $0<y<3$, $f_Y(y) = 0$ otherwise.

One last comment: X, Y not indep because $f_{X,Y}(x,y)$ not defined in rectangles, but also not independent since $f_{X,Y}(x,y) \ne f_X(x)f_Y(y)$ i.e. $\frac{2}{9} \ne \frac{2}{9}(3-x)\frac{2}{9}(3-y)$.