Caveat about independent versus dependent continuous random variables:

<u>Example</u>: Say X, Y have joint density $f_{X,Y}(x,y) = 24xy$

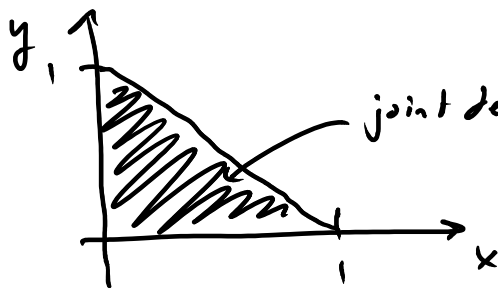
for $x \geq 0$, $y \geq 0$

and $x + y \leq 1$

$f_{X,Y}(x,y) = 0$ otherwise.

Initially we might think $f_{X,Y}(x,y)$ can be factored so X, Y independent

<u>BUT</u> factoring doesn't work for all pairs x, y in this example.



joint density is nonzero in this region.

E.g. if $X = \frac{3}{4}$ then $0 \leq Y \leq \frac{1}{4}$

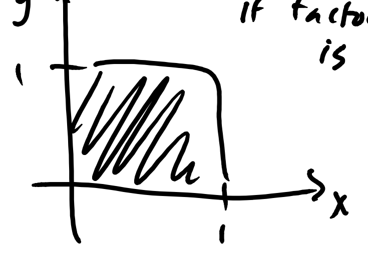
but with no prior information about X, Y can be between 0 and 1.

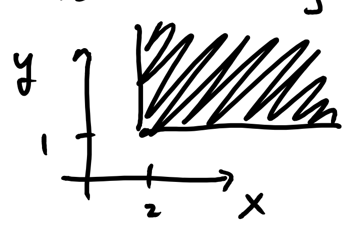
So knowing X affects the possible values of Y,

So in fact X and Y are dependent, not independent.

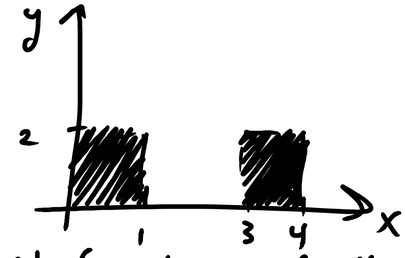
More generally, for X and Y to be independent, the joint density needs to factor, e.g. $f_{X,Y}(x,y) = (x \text{ stuff}) \cdot (y \text{ stuff})$ and for this factoring to occur completely, need $f_{X,Y}(x,y)$ to be nonzero on a grid of rectangular shaped regions (finite or infinite), for instance:

if factoring, things are OK (independence) if joint density is defined in a region like this





or



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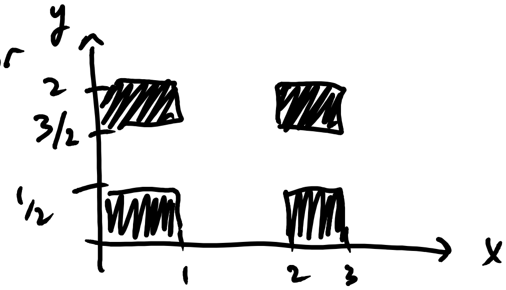

Etc., etc. Need a grid of rectangles for the domain of the joint density, if X and Y are to be independent.