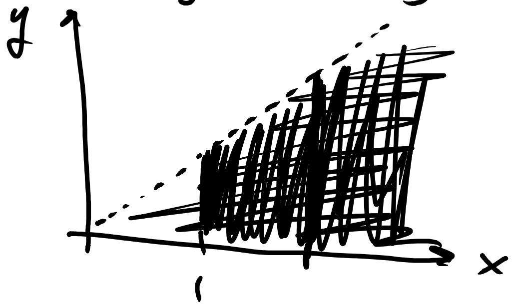
One more example: Suppose X, Y have joint density

$$f_{X,Y}(x,y) = \begin{cases} 9e^{-3x} & \text{when } 0<y<x \\ 0 & \text{otherwise} \end{cases}$$

Notice the joint density is defined here:



Find $P(X>1)$.

Two methods!

$$P(X>1) = \int_1^\infty \int_0^x 9e^{-3x}\,dy\,dx$$

$$= \int_1^\infty 9e^{-3x} \cdot y \Big|_{y=0}^{x} dx$$

$$= \int_1^\infty 9xe^{-3x}\,dx$$

use int. by parts

$u = 9x \qquad du = 9dx$

$dv = e^{-3x}dx \qquad v = \frac{e^{-3x}}{-3}$

$$= (9x)\left(\frac{e^{-3x}}{-3}\right)\Big|_{x=1}^{\infty} - \int_1^\infty (9)\left(\frac{e^{-3x}}{-3}\right)dx$$

$$= 9\cdot 1\cdot \frac{e^{-3}}{3} + \int_1^\infty 3e^{-3x}\,dx$$

$$= 3e^{-3} + \frac{3e^{-3x}}{-3}\Big|_{x=1}^{\infty}$$

$$= 3e^{-3} + e^{-3}$$

$$= 4e^{-3} \approx .199$$

i.e. about 20% of the time, X is larger than 1.

Method 2 First get $f_X(x)$.

$$f_X(x) = \int_0^x 9e^{-3x}\,dy$$

$$= 9e^{-3x} \cdot y \Big|_{y=0}^{x}$$

$$= 9xe^{-3x}$$

Now I know density of X is $f_X(x) = \begin{cases} 9xe^{-3x} & x>0 \\ 0 & \text{otherwise} \end{cases}$

$$P(X>1) = \int_1^\infty 9xe^{-3x}\,dx = (9x)\left(\frac{e^{-3x}}{-3}\right)\Big|_{x=1}^{\infty} - \int_1^\infty 9\,\frac{e^{-3x}}{-3}\,dx$$

$= \ldots =$ exactly same steps $= 4e^{-3} \approx .199.$

Need u-sub

$u = 9x \qquad du = 9dx$

$dv = e^{-3x}dx \qquad v = \frac{e^{-3x}}{-3}$

You can use whichever method makes you more comfortable.