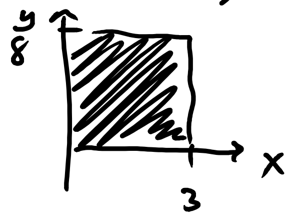
Now consider joint densities that are constant.

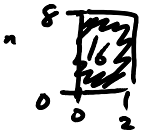
Nice fact from calculus: If I integrate a constant function over a region, the integral just equals that constant value times the area of the region.

For example: Say X and Y have constant joint density $\frac{1}{24}$ on the region where $0 \le x \le 3$, and $0 \le y \le 8$.



Find $P(X \le 2) = \int_0^2 \int_0^8 \frac{1}{24}\, dy\, dx = \int_0^2 \frac{8}{24}\, dx = \frac{16}{24}$.

Same as taking $\frac{1}{24}$ · area of region



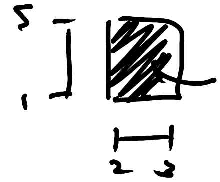
$= \boxed{\frac{16}{24}}$.

No need to integrate.

Find $P(2 \le X \le 3,\ 1 \le Y \le 5) = \int_2^3 \int_1^5 \frac{1}{24}\, dy\, dx$

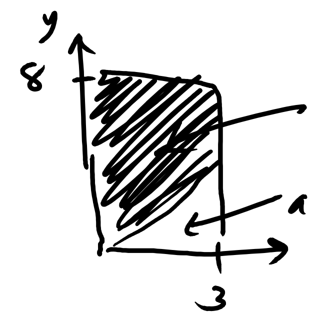
$= \int_2^3 \frac{4}{24}\, dx = \frac{4}{24}$

Easier way:
area of region



so $\boxed{\frac{4}{24}}$

Find $P(X < Y) = \int_0^3 \int_x^8 \frac{1}{24}\, dy\, dx = \int_0^3 \frac{8-x}{24}\, dx$



area $24 - 4.5 = 19.5 = \frac{39}{2}$

area $\frac{3 \cdot 3}{2} = \frac{9}{2} = 4.5$

$= \left. \frac{8x - x^2/2}{24} \right|_{x=0}^{3}$

$= \frac{24 - 9/2}{24}$

$= 1 - \frac{9}{48} = 1 - \frac{3}{16} = \frac{13}{16}$.

Easier way??

So constant for the joint density $\frac{1}{24}$

times the area of the region, $\frac{39}{2}$

multiply $\left(\frac{1}{24}\right)\left(\frac{39}{2}\right) = \frac{13}{8 \cdot 2} = \frac{13}{16}$. No integration needed.

Main point: If your joint density is constant, you are just integrating a constant over a region, so you get the constant times the area of the region! No need to integrate.