Jointly distributed continuous random variables

This is just a fancy way of saying that we want to analyze two (or more) continuous random variables at the same time.

Define $f_{X,Y}(x,y)$ to be the joint probability density function (joint density). It has the property that

$$P(X \in A,\ Y \in B) = \int_A \int_B f_{X,Y}(x,y)\,dy\,dx$$

For example, $P(0 \le X \le 2,\ 1 \le Y \le 7.5) = \int_0^2 \int_1^{7.5} f_{X,Y}(x,y)\,dy\,dx$

Also have a joint CDF (joint cumulative distribution function)

$$F_{X,Y}(a,b) = P(X \le a,\ Y \le b) = \int_{-\infty}^{a} \int_{-\infty}^{b} f_{X,Y}(x,y)\,dy\,dx$$

We get the joint CDF evaluated at (a,b) by integrating $f_{X,Y}(x,y)$, the joint density, from $-\infty$ to a with respect to x and by integrating from $-\infty$ to b with respect to y.

If we have the joint CDF $F_{X,Y}(x,y)$ of X and Y, we can get the joint density be differentiating with respect to each of the random variables: $f_{X,Y}(x,y) = \frac{d}{dx}\frac{d}{dy}F_{X,Y}(x,y) = \frac{d}{dy}\frac{d}{dx}F_{X,Y}(x,y)$ (either order is OK).