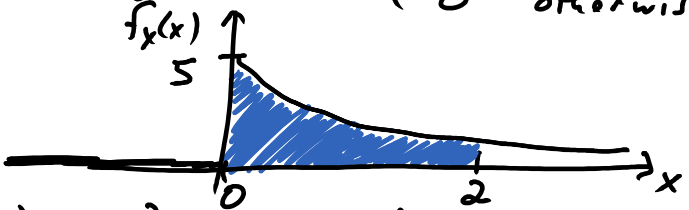
Relationship between the probability density function (density) and the cumulative distribution function (CDF)

With discrete random variables, we had no densities, but we had masses, and we sum the mass values $\leq a$ to get CDF $F_X(a) = P(X \leq a)$.
With continuous random variables, we have no masses, just densities, and we integrate the density $\leq a$ to get CDF $F_X(a) = P(X \leq a)$.

Picture of the density $f_X(x) = \begin{cases} 5e^{-5x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$



Find $P(0 \leq X \leq 2) = \int_0^2 f_X(x)\,dx = \int_0^2 5e^{-5x}\,dx = 1 - e^{-10}$

This corresponds to the area under the curve.
In general, the probability a random variable is in some range is equal to the area under the density curve in that range.

$F_X(a) = \int_{-\infty}^{a} f_X(x)\,dx.$ If we have the density, this is how we find the CDF: just integrate the density from $-\infty$ to the desired value, say "a".

In contrast, if we have the CDF only, but we want to know the density, we differentiate the CDF to get the density.

$\frac{d}{dx} F_X(x) = f(x)$ $\qquad$ $\frac{d}{da} F_X(a) = f_X(a).$

For example, we already calculated $F_X(a) = \begin{cases} 1 - e^{-5a} & \text{for } a > 0 \\ 0 & \text{otherwise} \end{cases}$

differentiate, $\frac{d}{dx}(1 - e^{-5x}) = -(e^{-5x})(-5)$
$= 5e^{-5x}$ i.e. get back the density function from the CDF.