Cumulative distribution function $F_X(x) = P(X \le x)$

$F_X(a) = P(X \le a)$

With continuous random variables, $F_X(a) = P(X \le a) = \int_{-\infty}^{a} f_X(x)\,dx$

For example, if $f_X(x) = 3$ for $0 \le x \le \frac{1}{3}$

$= 0$ otherwise

$F_X(a) = P(X \le a) = 0$ for $\underline{a < 0}$. Why: $\int_{-\infty}^{a} f_X(x)\,dx = \int_{-\infty}^{a} 0\,dx = 0$

$F_X(a) = P(X \le a) = 1$ for $\underline{a > \frac{1}{3}}$. Why: $\int_{-\infty}^{a} f_X(x)\,dx = \int_{-\infty}^{0} f_X(x)\,dx + \int_{0}^{1/3} f_X(x)\,dx + \int_{1/3}^{a} f_X(x)\,dx$

$= \int_{-\infty}^{0} 0\,dx + \int_{0}^{1/3} 3\,dx + \int_{1/3}^{a} 0\,dx$

$= 0 + 1 + 0$

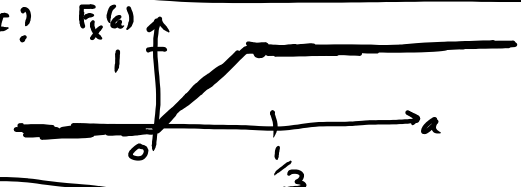
$= 1$

For "a" in the interesting region: $0 \le a \le 1/3$

$F_X(a) = P(X \le a) = \int_{-\infty}^{a} f_X(x)\,dx = \int_{-\infty}^{0} 0\,dx + \int_{0}^{a} 3\,dx$

$= 0 + 3a$

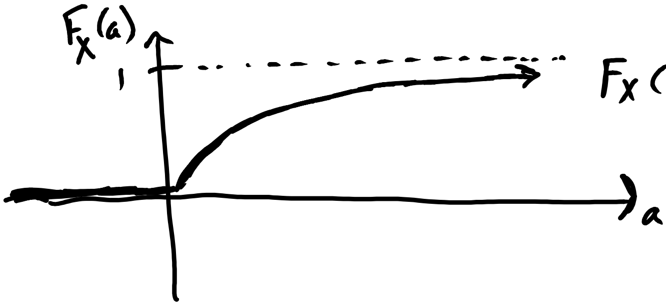
What is the graph of the CDF?



Other example: Say X has density $f_X(x) = \begin{cases} 5e^{-5x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$

$f_X(x) \Rightarrow$

How does the CDF look?



CDF: For $a \le 0$: $F_X(a) = \int_{-\infty}^{a} f_X(x)\,dx = \int_{-\infty}^{a} 0\,dx = 0$

For $a > 0$:

$F_X(a) = \int_{-\infty}^{a} f_X(x)\,dx = \int_{-\infty}^{0} 0\,dx + \int_{0}^{a} 5e^{-5x}\,dx$

$= 0 + \left. \frac{5e^{-5x}}{-5} \right|_{x=0}^{a}$

$= \boxed{1 - e^{-5a}}$