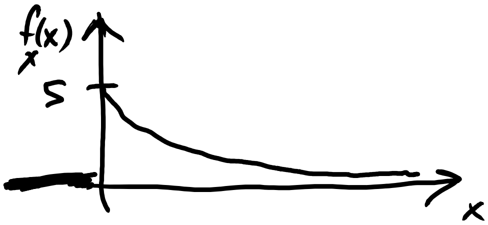
<u>Example</u> Say X has density $f_X(x) = \begin{cases} 5e^{-5x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$



Find $P(X > 2) = \int_2^\infty 5e^{-5x}\,dx$

$= \left.\frac{5e^{-5x}}{-5}\right|_{x=2}^{\infty}$

$= e^{-10}$

Find $P(X \leq 2) = \int_{-\infty}^{2} f_X(x)\,dx$

$= \int_{-\infty}^{0} f_X(x)\,dx + \int_0^2 f_X(x)\,dx$

$= \cancel{\int_{-\infty}^{0} 0\,dx} + \int_0^2 5e^{-5x}\,dx$

$= \left.\frac{5e^{-5x}}{-5}\right|_{x=0}^{2}$

$= 1 - e^{-10}$

Find $P(X = 2) = \int_2^2 5e^{-5x}\,dx = \left.\frac{5e^{-5x}}{-5}\right|_{x=2}^{2} = e^{-10} - e^{-10} = 0.$

This is a special case of a much more general phenomenon.
For any continuous random variable X and any density $f_X(x)$ function that defines such an X, we have $P(X = a) = 0$ for any a. Why?

$P(X = a) = \int_a^a f_X(x)\,dx = 0.$

This is different from the behavior with discrete random variables.
It gives us some freedom in the way we write inequalities:

$P(2 \leq X \leq 5) = \int_2^5 f_X(x)\,dx \longleftrightarrow P(2 < X < 5) = \int_2^5 f_X(x)\,dx$

difference is $P(X=2) = 0$, $P(X=5) = 0$

So we get the same answers either way.

Point of this:

$P(2 \leq X \leq 5) = P(2 < X < 5) = P(2 \leq X < 5) = P(2 < X \leq 5)$

These are all exactly the same values, all are $\int_2^5 f_X(x)\,dx$.

$P(X=2) = 0$ and $P(X=5) = 0$ so we drop or add boundaries as needed. No worries.